

## Practice Midterm

Mathematics 307

Spring 2008

- You have 50 minutes to complete the exam.
- A piece of notes (8.5 \* 11 inches, letter size, one side) will be allowed on the exam, along with a scientific (but not graphing) calculator.

**Problem 1.** Solve the differential equation

$$\frac{dy}{dt} + 3y = 4t^2e^{-3t} - te^{-2t}.$$

**Problem 2.** Find the solution of the initial value problem in explicit form

$$\frac{dy}{dt} = \frac{3t^2 - e^t}{2y - 8},$$

$$y(0) = 1.$$

(You do NOT need to determine the interval in which the solution exists.)

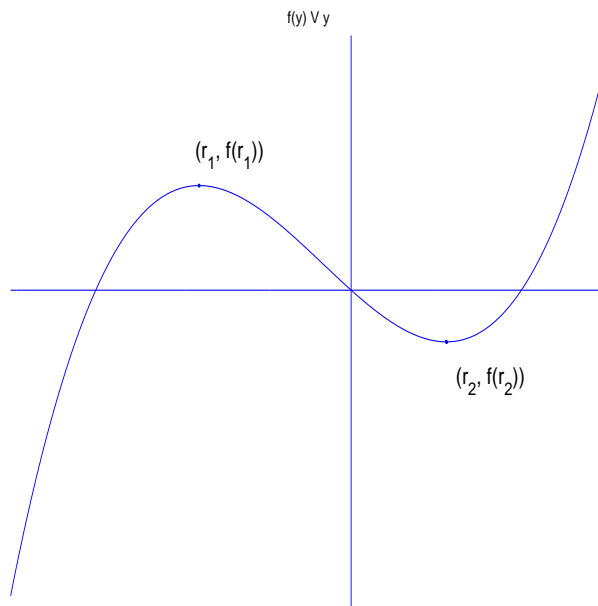
**Problem 3.** A tank contains 100 gal of water and 1 lb of salt. Water containing a salt concentration of  $(1 - e^{-\frac{1}{50}t})$  lb/gal flows into the tank at a rate of 2 gal/minute, and the well-stirred mixture in the tank flows out at the same rate. Find the amount of salt in the tank at any time.

**Problem 4.** Consider the differential equation

$$\frac{dy}{dt} = (y + 3)y(y - 2).$$

Determine the equilibrium solutions, classify each one as asymptotically stable, unstable or semistable, and sketch several graphs of solutions in the ty-plane.

(You might need the graph of  $f(y) = (y + 3)y(y - 2)$  below, where  $r_1 = \frac{-1 + \sqrt{19}}{3}$  and  $r_2 = \frac{-1 - \sqrt{19}}{3}$ .)



**Problem 5.** Consider the initial value problem

$$\frac{dy}{dt} = \cos(10\pi t + 5\pi y) - y, \quad y(0) = 0.$$

Use Euler's method with step size  $h = 0.1$  to approximate  $y(0.1)$  and  $y(0.2)$ .

**Problem 6.** Find the general solutions of the following differential equations.

(1)  $y'' + y' - 6y = 0$ .

(2)  $y'' + 4y' + 5y = 0$ .

**Problem 7.** Solve the initial value problem

$$y'' - 6y' + 9y = 0$$

$$y(0) = 2, \quad y'(0) = 2$$