Mid II Sol

1. a) $\boxed{2^x \ln 2}$ (10)

b) $f'(x) = \frac{1}{1+\left(\frac{x+8}{1-8x}\right)^2} \cdot \frac{(1-8x)+8(x+8)}{(1-8x)^2}$ (4)

$= \frac{65}{(1-8x)^2+(x+8)^2} = \boxed{\frac{1}{1+x^2}}$ (4) (2)

Recall: $(\arctan x)' = \frac{1}{1+x^2}$.

c) $f'(t) = \pi e^{\pi t}\sin \pi t + \pi e^{\pi t}\cos \pi t$ (5)

$f''(t) = \cancel{\pi^2 e^{\pi t}\sin \pi t} + \pi^2 e^{\pi t}\cos \pi t + \pi^2 e^{\pi t}\cos \pi t - \cancel{\pi^2 e^{\pi t}\sin \pi t}$

$= \boxed{2\pi^2 e^{\pi t}\cos \pi t}$ (5)

d) $\ln y = (1+\frac{1}{x})\ln x$ (5)

$\frac{1}{y} y' = \frac{-1}{x^2}\ln x + (1+\frac{1}{x})\frac{1}{x}$ (5)

$y' = \boxed{x^{(1+\frac{1}{x})}\left[\frac{-\ln x}{x^2} + \frac{x+1}{x^2}\right]}$ (5)

e) $\ln y = \ln(1+t) + \ln(1+t^3) - \frac{1}{4}\ln(1+t^2) - \frac{1}{4}\ln(1+t^4)$ (5)

$\frac{1}{y}\frac{dy}{dt} = \frac{1}{1+t} + \frac{3t^2}{1+t^3} - \frac{2t}{4(1+t^2)} - \frac{4t^3}{4(1+t^4)}$ (5)

$\frac{dy}{dt} = \boxed{\frac{(1+t)(1+t^3)}{\sqrt[4]{(1+t^2)(1+t^4)}}\left[\frac{1}{1+t} + \frac{3t^2}{1+t^3} - \frac{t}{2(1+t^2)} - \frac{t^3}{1+t^4}\right]}$ (5)

2. step1. At the y intercepts (0,1), (0,−1) (4)

$2x - y - x\frac{dy}{dt} + 2y\frac{dy}{dt} = 0$ (6)

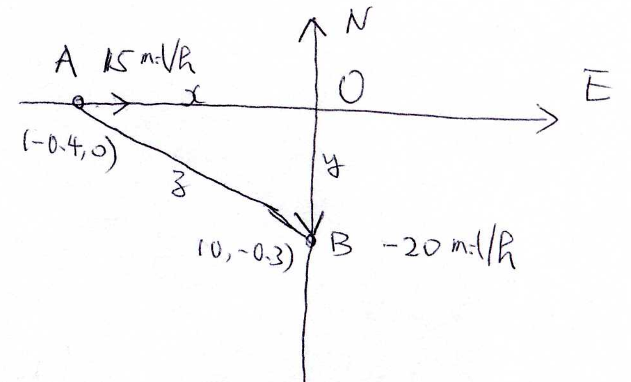
$\frac{dy}{dx} = \frac{1}{2}$ (4)

Step 2. Equations

$y - 1 = \frac{1}{2}x$ $\boxed{y = \frac{1}{2}x + 1}$

$y + 1 = \frac{1}{2}x$ $\boxed{y = \frac{1}{2}x - 1}$ (6)

3.



Let north & east be the positive direction

$OA = x$

$OB = y$

$|AB| = z$

$z^2(t) = x^2(t) + y^2(t)$ (10)

$z\frac{dz}{dt} = x\frac{dx}{dt} + y\frac{dy}{dt}\Big|_{\substack{x=-0.4,\ \frac{dx}{dt}=15 \\ y=-0.3,\ \frac{dy}{dt}=-20}}$

$= 0$

$\frac{dz}{dt} = 0$ mil/h

Answer: Car A & B are approaching each other at 0 mil/h. (10