M125 Midterm I Sols Oct. 16 08

1. A: $\frac{1}{\sqrt{x^4+1}+1}(x^4+1)' = \boxed{\frac{4x^3}{\sqrt{x^4+2}}}$ (5) (4) (1)

E/F: $\frac{1}{\sqrt{x^5+1}+1}(x^5+1)' = \boxed{\frac{5x^4}{\sqrt{x^5+2}}}$

2. i. A: $u = \ln x$ (3)

$du = \frac{1}{x}dx$ (3)

$\int_{\ln 1}^{\ln 3} u^3\,du = \frac{1}{4}u^4\Big|_0^{\ln 3} = \boxed{\frac{1}{4}(\ln 3)^4}$ (2) (2) (5)

E/F: ...

$\int_{\ln 1}^{\ln 3} u^2\,du = \frac{1}{3}u^3\Big|_0^{\ln 3} = \boxed{\frac{1}{3}(\ln 3)^3}$

ii. A: $u = 1+\sqrt{t}$ (5)

(5) $du = \frac{1}{2}\frac{1}{\sqrt{t}}dt$

$2du = \frac{1}{\sqrt{t}}dt$

$\int 2\cos u\,du = 2\sin u + C$

$= \boxed{2\sin(1+\sqrt{t}) + C}$ (5)

E/F ...

$\int 2\sin u\,du = -2\cos u + C$

$= \boxed{-2\cos(1+\sqrt{t}) + C}$

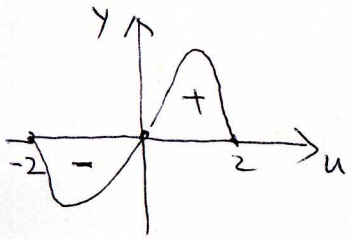
3. $u = x-2$ $u=2, x=4$; $x = u+2$

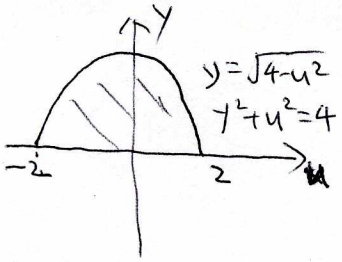
(5) $du = dx$ $u=-2, x=0$

$\int_{-2}^{2} (u+2)\sqrt{4-u^2}\,du$

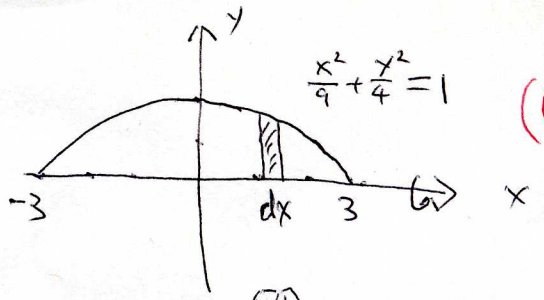
(5) $= \int_{-2}^{2} u\sqrt{4-u^2}\,du + 2\int_{-2}^{2}\sqrt{4-u^2}\,du$

$= \underset{(5)}{0} + \underset{(5)}{\pi\cdot 2^2} = \boxed{4\pi}$





4. (6)



vol of cylinder $= \pi R^2\,dx$

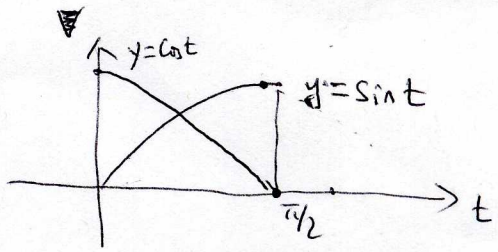
$= \pi y^2\,dx$ (8)

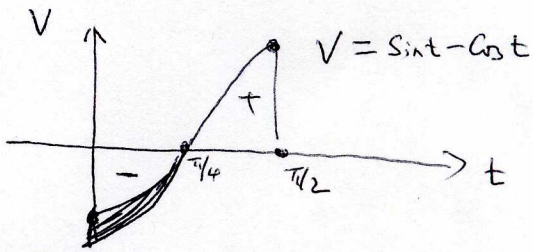
$= \pi\,4(1-\frac{x^2}{9})\,dx$

$vol = \int_{-3}^{3} 4\pi(1-\frac{x^2}{9})\,dx = 4\pi(x-\frac{x^3}{27})\Big|_{-3}^{3}$

$= 2\cdot 4\pi(3-1) = \boxed{16\pi}$ (6)

5.





a. displacement $= \int_0^{\frac{\pi}{2}} (\sin t - \cos t)\,dt$ (10)

$= (-\cos t - \sin t)\Big|_0^{\frac{\pi}{2}} = (-1) - (-1) = 0$ (5)

b. distance $= \int_0^{\frac{\pi}{2}} |\sin t - \cos t|\,dt$ (2)

$= \int_0^{\frac{\pi}{4}} (\cos t - \sin t)\,dt + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\sin t - \cos t)\,dt$ (1) (1)

$= 2\int_0^{\frac{\pi}{4}} (\cos t - \sin t)\,dt = 2(\sin t + \cos t)\Big|_0^{\frac{\pi}{4}}$

$= \boxed{2(\sqrt{2}-1)}$ (1)