Introduction to PDE M557/8/9

2016 Fall quarter (Instructor: Yuan)

Various ways of representing solutions to various PDEs: Fundamental solutions. Mean value formula (for Laplace and heat equations). Maximum principle. Liouville theorem. Analyticity. Self-similar solutions. Hopf-Cole transformations. Potential functions. Hodograph, Legendre, and Lewy transformations. Lewy's counterexample.

2017 Winter quarter (Instructor: Robin Graham)

I. Cauchy Problem: Formulation of Cauchy problem, characteristic and non-characteristic hypersurfaces, Cauchy-Kowalewsky and Holmgren Theorems. Cauchy problem for real first order equations. Cauchy problem for the wave equation: explicit solution via spherical means, fundamental solution, Huygen's phenomenon, Duhamel's principle.

II. Dirichlet Problem: Green's function and Poisson kernel for the Laplacian on the ball and half-space. L^2 Sobolev spaces on R^n and on bounded domains. Dirichlet problem for second order linear elliptic equations via Garding's inequality and theory of compact/Fredholm operators. Eigenfunction expansions. Higher interior and boundary regularity. Neumann problem.

2017 Spring quarter (Instructor: Yuan)

Qualitative theory for parabolic PDEs: Existence of weak solutions. Regularity. Maximum principle. Harnack inequality. Variational and non-variational techniques for nonlinear PDEs: Existence of minimizers. Regularity. Constraints. Critical points. Monotonicity method. Fixed point methods. Sub/super solutions. Nonexistence. Moving plane method. Viscosity solutions.

Fall 2016 and Spring 2017 Textbook: Partial differential equations, L. C. Evans. Reference: PDE, F. John

Winter 2017 References/Textbooks: Gerald B. Folland, Introduction to Partial Differential Equations; Michael E. Taylor, Partial Differential Equations Basic Theory.