Theory of Linear and Nonlinear Second Order Elliptic Equations

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M583E Spring19, MW 2:30–3:50PM DEN210
M581A Fall19, MW 9–10:20AM PDL C-401

Linear Theory: Solvability, a priori estimates, Schauder and Calderon-Zygmund estimates, and regularity

Nonlinear Theory: De Giorgi Nash Moser theory for divergence equations (eg. minimal surface equation), Krylov-Safonov theory for nondivergence equations (eg. Monge-Ampere equation, special Lagrangian equations, Bellman equations, and Isaacs equations).

Contents: Harmonic functions (properties), Schauder for $\triangle$, Weighted norm, solvability for Laplace, Boundary Schauder, L^p for $\triangle$, Energy method, capacity, Poincare, Soblev, $W^{1,2}$ or H^1 space, trace; De Giorgi/Nash, Harnack, Quick applications of Harnack, Minimal surface equations, Viscosity solutions to Nondivergence equations, Alexandrov maximum principle, Krylov-Safonov, Uniqueness and Existence of viscosity solutions, $C^{1,\alpha}$ regularity, $C^{2,\alpha}$ regularity for convex equations, Monge-Ampere and special Lagrangian equations, Bellman equations, and Isaacs equations.

Prerequisites: Advanced calculus/Real Analysis (Comparable to Rudin's Principles of Mathematical Analysis; Previous PDE knowledge is welcome, but not required)

References:

Han, Qing; Lin, Fanghua, Elliptic partial differential equations. Second edition. Courant Lecture Notes in Mathematics, 1. Courant Institute of Mathematical Sciences, New York; ; American Mathematical Society, Providence, RI, 2011.

Caffarelli, Luis A.; Cabre, Xavier, Fully nonlinear elliptic equations. American Mathematical Society Colloquium Publications, 43. American Mathematical Society, Providence, RI, 1995.

Gilbarg, David; Trudinger, Neil S. Elliptic partial differential equations of second order. Reprint of the 1998 edition. Classics in Mathematics. Springer-Verlag, Berlin, 2001.

Lecture notes will be provided.