Elliptic PDE Fall 2013 HW2

1. Recall that for harmonic function u

$$u(0) = \frac{1}{|\partial B_1|} \int_{\partial B_1} u(y)\, dA_y.$$

Is the mean value property still true for the boundary of a cube instead of the boundary of a ball?

2. Let $u \in L^1$ and fix r, assume

$$u(x) - u(x_0) = \frac{1}{|B_r|} \left[\int_{B_r(x)} u(y)\, dy - \int_{B_r(x_0)} u(y)\, dy \right].$$

Show that the right hind side goes to 0 as x goes to x_0.

3. Let $\psi_\varepsilon = \frac{1}{\varepsilon^n} \psi\left(\frac{x}{\varepsilon}\right)$, where ψ is a smooth approximation to the characteristic function $\chi_{B_r(x)}$ (and $\psi \in C_0^\infty$). Let $u \in \mathcal{D}$ (or just L^1) such that $u * \psi_\varepsilon$ is independent of ε. Show that $u = u * \psi_\varepsilon$ in the distribution sense.

4. Let

$$u(x) = \frac{1}{|\partial B_1|} \int_{\partial B_1} \frac{\left(1 - |x|^2\right)}{|y - x|^n} \varphi(y)\, dA_y \quad \text{for } \varphi \in C^0(\partial B_1),$$

show that $\triangle u(x) = 0$ and $\lim\limits_{x \in B_1 \to y} u(x) = \varphi(y)$ for $|y| = 1$.