Elliptic PDE Fall 2013 HW3

1. Show that for $u \in C^0$, the subharmonicity $\triangle u \geq 0$ in the distribution/IBP sense and the subharmonicity $\triangle u \geq 0$ in the viscosity sense are equivalent.

2. Show that

$$\begin{cases} \triangle p = q \quad q \in \mathcal{P}^k \\ p = 0 \text{ on } \partial B_1 \end{cases}$$

always has a polynomial solution $p \in \mathcal{P}^{k+2}$, where $\mathcal{P}^k$ represents the set of polynomials of degree at most k.

3. Show that

$$\begin{cases} \triangle p = q \quad q \in \mathcal{P}^k \\ p = 0 \text{ on } \partial\Omega \end{cases}$$

always has a polynomial solution $p \in \mathcal{P}^{k+2}$, where $\partial\Omega = \left\{ x : \sum_{i=1}^{n} x_i^4 = 1 \right\}$.