

Elliptic PDE Fall 2013 HW5

1. Given the Poisson kernel $P_y(x) = c_n \frac{y}{|(x, y)|^n}$ with $(x, y) \in R^{n-1} \times R^1$. Let $g \in C^0(R^{n-1}) \cap L^\infty(R^{n-1})$. Show that the convolution

$$u(x, y) = P * g = \frac{2}{|\partial B_1|} \int_{R^{n-1}} \frac{y}{|(\xi - x, y)|^n} g(\xi) d\xi$$

is harmonic in the upper half space $y > 0$ and takes boundary value $g(x)$ as $y \rightarrow 0^+$.

2. Show that

$$\|P_y * g\|_{C^\alpha(\overline{R_+^n})} \leq C(n) \|g\|_{C^\alpha(R^{n-1})}.$$

3. Show that harmonic functionS of (x, y)

$$\begin{aligned} \frac{x}{|(x, y)|^n} &= \lim_{\substack{R \rightarrow \infty \\ \varepsilon \rightarrow 0}} \frac{2}{|\partial B_1|} \int_{(B_R \setminus B_\varepsilon) \cap R^{n-1}} \frac{y}{|(x - \xi, y)|^n} \frac{\xi}{|\xi|^n} d\xi \\ &= P_y * \frac{x}{|x|^n}. \end{aligned}$$