Erratum to

Symmetric Markov Processes, Time Change, and Boundary Theory

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Princeton University Press, 2012

(Last update: *April 11, 2021*)

- p.15, line 8: There is a gap in the proof of $C_b(F) \cap L^2(F;m)$ being dense in $L^2(F;m)$. Here we show the part (ii) of Lemma 1.1.14 directly that $\{T_t; t \geq 0\}$ is a strongly continuous semigroup on $L^2(E;m)$.

  Since m is σ-finite, there is a partition $\{E_k; k \geq 1\}$ of F so that $m(E_k) < \infty$ for every $k \geq 1$. Since every $f \in L^2(F;m)$ can be L^2-approximated by a sequence of simple functions in $L^2(F;m)$, it suffices to show that for any $A \subset F$ having $m(A) < \infty$, $T_t 1_A$ converges to 1_A in $L^2(F;m)$ as $t \to 0$. For simplicity, denote $m|_{E_j}$ by m_j. Since each m_j is a regular measure, for any $\varepsilon > 0$, there a compact set $K_j \subset A$ and an open set $U_j \supset A$ so that $m_j(A \setminus K) < \varepsilon/2^j$ and $m_j(U_j \setminus A) < \varepsilon/2^j$. Since $m(A) = \sum_{j=1}^\infty m_j(A)$, there is some $N \geq 1$ so that $\sum_{j=N+1}^\infty m_j(A) < \varepsilon/2$. Define $K = \cup_{j=1}^N K_j$. Then $K \subset A$, K is compact,

  $$m(A \setminus K) \leq \sum_{j=1}^N m_j(A \setminus K_j) + \sum_{j=N+1}^\infty m_j(A) < \varepsilon, \tag{1}$$

  and

  $$m(\cap_{j=1}^\infty U_j \setminus A) \leq \sum_{j=1}^\infty m_j(U_j \setminus A) < \varepsilon. \tag{2}$$

  For each $j \geq M$, define

  $$g_j(x) = \frac{d(x, (\cap_{k=1}^j U_k)^c)}{d(x, (\cap_{k=1}^j U_k)^c)) + d(x, K)}.$$

Clearly $g_j \in C_b(F)$ with $0 \le g_j \le 0$ on F, $g_j = 1$ on K, and $g_j = 0$ on $(\cap_{k=1}^{j} U_k)^c$. Note that g_j is decreasing in j and $g_\infty(x) := \lim_{j\to\infty} g_j(x)$ vanishes on $(\cap_{k=\infty}^{j} U_k)^c$. Hence by (2),

$$\int_F 1_{A^c}(x) g_\infty(x)^2 m(dx) \le m(\cap_{k=1}^{\infty}(U_k \cap A^c) \le \sum_{k=1}^{\infty} m_k(U_k \setminus A) < \varepsilon.$$

Thus by the monotone convergence theorem, there is some $N_1 \ge N$ so that

$$\int_F 1_{A^c}(x) g_{N_1}(x)^2 m(dx) < \varepsilon.$$

Hence by the L^2-contractiveness of $\{T_t; t \ge 0\}$ and the Cauchy-Schwartz inequality,

$$\begin{aligned}
& \limsup_{t\to 0} \|1_A g_{N_1} - T_t(1_A g_{N_1})\|^2_{L^2(F;m)} \\
\le\ & 2\|1_A g_{N_1}\|^2_{L^2(F;m)} - 2\liminf_{t\to 0} \int_F 1_A g_{N_1} T_t(1_A g_{N_1}) m(dx) \\
=\ & 2\|1_A g_{N_1}\|^2_{L^2(F;m)} - 2\liminf_{t\to 0} \left(\int_F 1_A g_{N_1} P_t g_{N_1} m(dx) - \int_F 1_A g_{N_1} T_t(1_{A^c} g_{N_1}) m(dx) \right) \\
\le\ & 2m(A)^{1/2} \|1_{A^c} g_{N_1}\|_{L^2(F;m)} < 2\sqrt{m(A)\varepsilon}.
\end{aligned}$$

On the other hand, as by (1),

$$\|1_A - 1_A g_{N_1}\|_{L^2(F;m)} \le m(A \setminus K)^{1/2} \le \varepsilon^{1/2},$$

we have

$$\begin{aligned}
\limsup_{t\to 0} \|1_A - T_t 1_A\|_{L^2(F;m)} &\le 2\varepsilon^{1/2} + \limsup_{t\to 0} \|1_A g - T_t(1_A g_{N_1})\|_{L^2(F;m)} \\
&\le 2\varepsilon^{1/2} + 2(m(A)\varepsilon)^{1/4}.
\end{aligned}$$

Since $\varepsilon > 0$ is arbitrary, this shows that $\limsup_{t\to 0} \|1_A - T_t 1_A\|_{L^2(F;m)} = 0$.

- p.27, line 13: ‘countable base’ should be ‘a countable base’.
- p.27, line 16: ‘(ii).’ should be ‘(ii′)’.
- p.40, line -19: S_n should be S_{nh}.

- p.42, line 15: '$\to$' should be '$=$'
- p.44, line -7: the first (2.1.20) should be (2.1.19).
- p.44, line -6: '$G(\eta - \eta G^\eta \eta)$' should be '$G_\alpha(\eta - \eta G^\eta \eta)$'
- p.110, line 13: '$\{p_B^1 > 0\}$' should be '$\{p_B^1 < 1\}$'
- p.110, line 16: '$\{p_B^1 \geq \frac{1}{k}\}$' should be '$\{p_B^1 \leq 1 - \frac{1}{k}\}$'
- p.110, line 20: '$p_B^1(X_\sigma) = 0$' should be '$p_B^1(X_\sigma) = 1$'
- p.143, line -2: the phrase "In view of Theorem 3.5.4" should be replaced by "In view of Lemma 1.3.15 and Theorem 3.1.4".
- p.206, line 12: delete 'F'.
- p.219, lines 4 and 5: 'dt' should be 'dr' (two places)
- p.219, Theorem 5.5.9: $(U(dx, dy) + J(dx, dy))$.
- p.376, in $(\mathbf{M}^\circ.\mathbf{3})$: '$\sup_{x\in V} G_1^0 \varphi^{(i)}(x) < \infty$' should be '$\inf_{x\in V} G_1^0 \varphi^{(i)}(x) > 0$'.
- p.384, line 12: '$H^1(D)$' should be '$H_e^1(D) \cap L^2(D; m)$'.
- p.474, line 9: 'holds' should be 'holes'.