

Final Exam–Math 124 C/D, Spring 2007

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You may use a simple scientific calculator and one page of two-sided handwritten notes (standard 8.5×11 sheet). Graphing calculators are not allowed on exams.

IV Story problems: (a) Related rates problems. (b) Optimization problems. (c) Estimation/approximation. (d) Parametric equations.

Example 1: The volume of a cube is increasing at a rate of $10 \text{ cm}^3/\text{min}$. How fast is the surface area increasing when the length of an edge is 30 cm ?

Example 2: A right circular cylinder is inscribed in a sphere of radius r . Find the largest possible surface area of such a cylinder.

Example 3: A particle is moving in the plane according to the parametric equations

$$x = \cos t + 2t, y = \sin t$$

where t is time and $t \geq 0$.

(a) Find the horizontal velocity $\frac{dx}{dt}$ and vertical velocity $\frac{dy}{dt}$.

(b) Using the chain rule, find $\frac{dy}{dx}$ at all times t .

(c) Find an equation of the tangent line to the path at the point $(x(\pi), y(\pi))$.

Example 4: The angle of elevation of the Sun is decreasing at a rate of 0.25 rad/h . How fast is the shadow cast by a 400-ft -tall building increasing when the angle of elevation of the Sun is $\pi/6$?

Example 5: A cone-shaped paper drinking cup is to be made to hold 27 cm^3 of water. Find the height and the radius of the cup that will use the smallest amount of paper.

Example 6: Suppose $y = f(x)$ is a function that satisfies the equation

$$x^4 + 3xy + y^4 = 5$$

and passes through the point $(1, 1)$. Use linear approximation to estimate the y value of a nearby point on the curve with $x = 1.02$.

Example 7: You are on a ship heading North at $12 \text{ miles per hour}$ toward an island when you spot a sailboat heading East away from the same island. When your ship is 9 miles from the island, the angle measured clockwise from due North to the boat is seen to be $\frac{\pi}{6}$ radians and the angle is increasing at $\frac{\pi}{3}$ radian/hour. How fast is the sailboat moving?

Example 8: Find the dimensions of the rectangle of largest area that has its base on the x -axis and its other two vertices above x -axis and lying on the parabola $y = 8 - x^2$.

V Others: (a) Continuity. (b) Differentiability, meanings of $f'(x)$ and $f''(x)$.

(c) Abstract theory, rules, etc. (d) Pre-Calculus stuff. (see old final exams)