

## HW1 is due Tuesday, April 8

HW1 (consisting of 12 problems) is due in quiz section on Tuesday week 2.

Reading homework: Read and understand the whole “Taylor Notes” by Tuesday April 8.

Problems 1-3 are based on the material in Section 1 of the Taylor Notes.

1. Find the first Taylor polynomial  $T_1(x)$  for  $f(x)$  based at  $b$  and use the Tangent Line Error Bound to bound the error  $|f(x) - T_1(x)|$  on the interval  $I$  where

- (a)  $f(x) = e^x$      $b = 0$      $I = [-1, 1]$ .
- (b)  $f(x) = \ln(1 + x)$      $b = 0$      $I = [-\frac{1}{2}, \frac{1}{2}]$ .
- (c)  $f(x) = \sin(x)$      $b = 0$      $I = [-0.1, 0.1]$ .
- (d)  $f(x) = x^{\frac{1}{3}}$      $b = -8$      $I = [-9, -7]$ .

2. For each function and base point, find the first Taylor polynomial based at  $b$  and then use the Tangent Line Error Bound to find an interval  $J$  containing  $b$  so that the error bound is at most 0.01 on  $J$ .

- (a)  $f(x) = \ln(x)$      $b = 1$ .
- (b)  $f(x) = \cos(x)$      $b = \frac{\pi}{6}$ .
- (c)  $f(x) = x^{\frac{1}{3}}$      $b = 8$ .

3. In math 124, we used the tangent line approximation to estimate a function. For example problem 50, page 269 in Stewart says:

*Suppose that we don't have a formula for  $g(x)$  but we do know that  $g(2) = -4$  and  $g'(x) = \sqrt{x^2 + 5}$  for all  $x$ . Use a linear approximation to estimate  $g(1.95)$  and  $g(2.05)$ .*

Use the Tangent Line Error Bound to bound the error in these two approximate values.

Problems 4-8 are based on the material in Sections 2 and 3 of the Taylor Notes.

4. Find the second Taylor polynomial  $T_2(x)$  for  $f(x)$  based at  $b$  and use the Quadratic Approximation Error Bound to bound the error  $|f(x) - T_2(x)|$  on the interval  $I$  where

- (a)  $f(x) = e^x$      $b = 0$      $I = [-1, 1]$ .
- (b)  $f(x) = \ln(1 + x)$      $b = 0$      $I = [-\frac{1}{2}, \frac{1}{2}]$ .
- (c)  $f(x) = \sin(x)$      $b = 0$      $I = [-0.1, 0.1]$ .
- (d)  $f(x) = x^{\frac{1}{3}}$      $b = -8$      $I = [-9, -7]$ .

5. For each function and base point, find the second Taylor polynomial based at  $b$  and then use the Quadratic Approximation Error Bound to find an interval  $J$  containing  $b$  so that the error bound is at most 0.01 on  $J$ . (Note that your intervals  $J$  here should be bigger than the answers you obtained to problem 2).

- (a)  $f(x) = \ln(x)$      $b = 1$ .
- (b)  $f(x) = \cos(x)$      $b = \frac{\pi}{6}$ .
- (c)  $f(x) = x^{\frac{1}{3}}$      $b = 8$ .

6. (a) Find the 5th Taylor polynomial for  $f(x) = x^5 - 3x^4 + 5x^2 + 6x - 2$  based at  $b = 1$ .

*Problem 6(a) gives a method for writing a polynomial in powers of  $(x - 1)$  instead of powers of  $x$ . It is sometimes useful if you are interested in the values of the polynomial near  $x = 1$ . Another technique would be to substitute  $u = x - 1$  or  $x = u + 1$ , multiply out the polynomial, collect together powers of  $u$  and then substitute back  $u = x - 1$ . Using Taylor polynomials is a bit easier.*

(b) If  $f$  is the polynomial in part (a), why is the  $n$ th Taylor polynomial equal to  $f$  whenever  $n \geq 5$ ?

*Hint: See Taylor's Inequality. Can you make a similar statement for any polynomial?*

7. Let  $f(x) = \ln(1 - x)$ .

(a) Find the  $n$ th Taylor polynomial  $T_n(x)$  for  $f(x)$  based at  $b = 0$ .

(b) Using Taylor's Inequality, find  $n$  so that  $|T_n(x) - f(x)| < 0.01$  on the interval  $[-0.5, 0.5]$ .

(c) On the smaller interval  $I = [-0.1, 0.1]$ , how close is  $T_n$  (from part (b)) to  $f(x)$ ?

8. Let  $f(x) = \sqrt{1 + x}$ .

(a) Find the  $n$ th Taylor polynomial  $T_n(x)$  for  $f(x)$  based at  $b = 0$ .

(b) Using Taylor's Inequality, find  $n$  so that  $|T_n(x) - f(x)| < 0.01$  on the interval  $[-0.5, 0.5]$ .

(c) On the smaller interval  $I = [-0.1, 0.1]$ , how close is  $T_n$  (from part (b)) to  $f(x)$ ?

Problems 9-12 are based on the material in Section 4 of the Taylor Notes.

9. Write the sum in expanded form:

(a)

$$\sum_{k=0}^6 \frac{2k - 1}{2k + 1}$$

(b)

$$\sum_{k=1}^7 \frac{1}{k} x^{2k}$$

(c)

$$\sum_{k=2}^7 k^2 x^k$$

10. Write the sum using the sigma notation.

(a)

$$\sqrt{3} + \sqrt{4} + \sqrt{5} + \sqrt{6} + \sqrt{7} + \sqrt{8}$$

(b)

$$1 + 2 + 4 + 8 + 16 + 32 + 64$$

(c)

$$1 + x + x^2 + x^3 + x^4 + x^5$$

11. (a) Write the following in the form

$$\sum_{k=0}^n a_k x^k$$

(i.e. combine and use the distributive law).

$$2 \sum_{k=1}^n \frac{x^k}{k} - 3 \sum_{k=1}^n 4^k x^k.$$

(b) What is the second Taylor polynomial for this function (based at  $b = 0$ )?

12. (a) Find

$$\frac{d}{dx} \sum_{k=0}^n x^k.$$

Express your answer using the sigma notation, and then in expanded form:  $a_0 + a_1x + a_2x^2 + \dots$ , where  $a_0, a_1, a_2$  are given explicitly (only the first three terms are explicit).

(b) Find

$$\int_0^t \left( \sum_{k=0}^n x^k \right) dx.$$

Express your answer in two ways as in part (a).

**HW is due on Tuesday**