Midterm one–Math 126 C/D, Winter 2017

Midterm one will be given on Thursday, Jan. 26 in quiz section. It will cover Sections 10.1, 12.1-12.6 and 13.1.

Some basic rules

1. You are allowed to use a TI-30X IIS calculator. But **NO** other calculators are allowed.
2. You are allowed to have one page of hand-written notes of standard size.
3. Make sure to show all your work. You will not receive any partial credit unless all work is clearly shown.
4. Unless otherwise stated, always give your answers in exact form. For example, 3π, $\sqrt{2}$, $\ln 2$ are in exact form, the corresponding approximations 9.424778, 1.4142, 0.693147 are not in exact form.
5. There are four questions in the exam. Each question contains several parts.

Practice problems

Problem 1: Computation concerning 3-d objects.

Example: Let $\overrightarrow{\mathbf{u}} = \langle 1, 2, 3\rangle$ and $\overrightarrow{\mathbf{v}} = \langle -2, -1, 7\rangle$.

(a) Find $\overrightarrow{\mathbf{u}} \bullet \overrightarrow{\mathbf{v}}$.
(b) Find $\overrightarrow{\mathbf{v}} \bullet \overrightarrow{\mathbf{u}}$.
(c) Find $\overrightarrow{\mathbf{u}} \times \overrightarrow{\mathbf{v}}$.
(d) Find $\overrightarrow{\mathbf{v}} \times \overrightarrow{\mathbf{u}}$.
(e) Find the two unit vectors parallel to $\overrightarrow{\mathbf{u}} \times \overrightarrow{\mathbf{v}}$.
(f) Find the vector projection of $\overrightarrow{\mathbf{u}}$ onto $\overrightarrow{\mathbf{v}}$.
(g) Find the vector projection of $\overrightarrow{\mathbf{v}}$ onto $\overrightarrow{\mathbf{u}}$.
(h) Is the cross product of two unit vectors always a unit vector?
(i) Find an equation of the plane that passes through $(1, 2, 3)$ and is parallel to the plane $-4x - 6y + z - 10 = 0$.

Problem 2: (Sections 10.1 and 13.1) Parametric curves in $\mathbf{R}^2$ or in $\mathbf{R}^3$. Practice problems: p. 642, #24, 28, p. 846, #21-26, 40-44.

Example: (a) Find a vector function that represents the curve of intersection of the paraboloid $z = 4x^2 + y^2$ and the cylinder $x^2 + y^2 = 1$.

(b) Two particles travel along the space curves

$$\overrightarrow{\mathbf{r}}_1(t) = \langle t, t^2, t^3 \rangle$$

$$\overrightarrow{\mathbf{r}}_2(t) = \langle 1 + 2t, 1 + 6t, 1 + 14t \rangle$$

Do the particles collide? Do they paths intersect?

Problem 3: (Sections 12.1-12.4) Other questions in 3-D: triple product, direction angles, angle formulas, area and volume formulas.

Practice problems: p. 799, #21, 23, 25, 37, 39, pp. 806-807, #23, 31, 33, 37, 39, 47, pp. 814-815, #3,5,7, 17, 33, 43.

Example: Let $\overrightarrow{\mathbf{u}} = \langle 4, 3, 0 \rangle$ and $\overrightarrow{\mathbf{v}} = \langle 5, 5, 5 \rangle$ and $\overrightarrow{\mathbf{w}} = \langle 2, 3, c \rangle$. (i) Find the angle between $\overrightarrow{\mathbf{u}}$ and $\overrightarrow{\mathbf{v}}$. (ii) Find the volume of the parallelepiped determined by the vectors $\overrightarrow{\mathbf{u}}, \overrightarrow{\mathbf{v}}, \overrightarrow{\mathbf{w}}$. (iii) Find c such that vectors $\overrightarrow{\mathbf{u}}, \overrightarrow{\mathbf{v}}, \overrightarrow{\mathbf{w}}$ are coplanar. (iv) Find c such that $\overrightarrow{\mathbf{u}} \times \overrightarrow{\mathbf{v}}$ is orthogonal to $\overrightarrow{\mathbf{w}}$.

Problem 4: (Sections 12.5-12.6) Lines, planes, cylinders, and quadric surfaces in $\mathbf{R}^3$, angle between two planes, distance from a point to a plane.

Practice problems: pp. 824-826, #7, 13, 23, 35, 45, 53, 65, 67, 70, 71.
pp. 833-834, #21-28, 31, 33, 43, 49, 50, and p. 836, #17, 23, 33, 35.

Example: (a) Consider the plane $x + y + z = 3$ and the line L_1: $\overrightarrow{\mathbf{r}}(t) = \langle t - 1, 2t, -t + 1 \rangle$. (i) Determine whether the plane and the line intersect or parallel. (ii) If intersect, find the point of intersection. If parallel, find the distance between them. (iii) Find a line L_2 on the plane such that L_1 and L_2 are intersect and orthogonal.

Example: A surface consists of all points P such that the distance from P to $(1, 0, -1)$ is twice the distance from P to the plane $y = 1$. Find an equation for this surface and identify it.

Note that problems in the exam may not be in this order and that practice problems will not occur in the exam.

Old Exam1 can be found at
http://www.math.washington.edu/∼m126/midterms/midterm1.php
Please note that some of these old exam1 cover Sections 10.2-10.3 and 13.2-13.3 that will not occur in this Exam.