Midterm Two–Math 126 C and D, Winter 2017

Midterm two will be given on Thursday, Feb. 16, 2017 in Quiz Section. It covers Ch. 10, 13 and 14.

Some basic rules

1. You are allowed to use a TI-30X IIS calculator. But **NO** other calculators are allowed.
2. You are allowed to have one page of hand-written notes of standard size.
3. Make sure to show all your work. You will not receive any partial credit unless all work is clearly shown.
4. Unless otherwise stated, always give your answers in exact form. For example, 3π, $\sqrt{2}$, $\ln 2$ are in exact form, the corresponding approximations 9.424778, 1.4142, 0.693147 are not in exact form.
5. There are four questions in the exam. Each question contains several parts.

Topics

Topic 1: (Sections 10.3+ 13.3) Polar coordinates, polar curve and tangent line.

Example: (1) Find an equation of the tangent line to the polar curve $r = 2 + \cos\theta$ at $\theta = \pi/3$.

(2) Consider the helix given by $\overrightarrow{r}(t) = \langle \sin(t), \cos(t), 3t \rangle$. Find the parametric equation of the tangent line to the helix at the point $(0, -1, 3\pi)$.

Topic 2: (Ch. 13) Parametric equations of 3-D curves, vector functions and space curves, derivatives and integrals, arclength, curvature and TNB system, the normal and osculating planes, velocity and acceleration, tangential and normal components. See pp. 874-875 for review exercises.

Examples: (1) Let $\overrightarrow{\mathbf{r}}(t) = \langle 2t + 7, \sin(t-2), \cos(t-2) \rangle$ be the position function.

(a) Find the velocity and acceleration.
(b) Find the curvature of the curve at any time t.
(c) Find TNB at any time t.
(d) Find the tangential and normal components of the acceleration.
(e) Find the normal plane and osculating plane at $t = 0$.

(2) The position function of a particle is given by

$$\overrightarrow{\mathbf{r}}(t) = \langle -3t, t^2 + 2, 1 + t - t^2 \rangle.$$

Let C be the corresponding curve determined by the above equation.

(i) Find the point(s) on the curve C where C has the maximum curvature

(ii) Compute the minimum speed of the particle.

Topic 3: (Chapter 14) Functions of two (or several) variables, surfaces and level curves (contour map), partial derivatives, higher order partial derivatives, tangent plane/linear approximation/differentials. See review exercises on pp. 968-970.

Examples:

(1) Let $f(x, y) = x\ln(y + x) + e^{xy} - 3x + 2y^2$. Find f_{xx}, f_{xy}, f_{yx} and f_{yy}.

(2) Let $f(x, y) = e^{\sin(x)-2y} + \tan(x^2 - 1)$. Find the tangent plane to the surface at $(0, 0)$. Use it to approximate $f(0.1, -0.2)$.

Topic 4: (Section 14.7) Critical points, maximum/minimum values, story problems, first derivative test, second derivative test, saddle points, $\rightarrow$ story problems $\leftarrow$.

(1) Find the points on the surface $2x^3y + z^2 = 4$ that are closest to the origin.

(2) Consider the function $f(x, y) = x^2 + y^2 - xy$ over D, where D is the region enclosed by the circle of radius 4 centered at the origin.

(a) Find and classify all critical points.

(b) Find the absolute maximum value of $f(x, y)$ over D.

(3) Find nonnegative numbers x, y and z that minimize the quantity

$$M = x^2 + y^2 + z^2$$

subject to the condition

$$xy^2z^4 = 1.$$

Note that problems in the exam may not be in this order and that practice problems will not occur in the exam.

Old Exam2 can be found at
http://www.math.washington.edu/~m126/midterms/midterm2.php