Final Exam–Math 126 C/D, Winter 2017

Final Exam will be given on **Saturday, March 11** from 5:00-7:50pm. It will cover Taylor Notes, Ch. 10, 12, 13, 14 and 15.

Suggestions: Use old final exams as practice tests.

http://www.math.washington.edu/∼m126/finals/final.php

Some basic rules

1. You are allowed to use a TI-30X IIS calculator. But **NO** other calculators are allowed.
2. You are allowed to have one page of hand-written notes of standard size.
3. Make sure to show all your work. You will not receive any partial credit unless all work is clearly shown.
4. Except for # 7, always give your answers in exact form. For example, 3π, $\sqrt{2}$, $\ln 2$ are in exact form, the corresponding approximations 9.424778, 1.4142, 0.693147 are not in exact form.
5. There are eight questions in the exam. Each question contains several parts.
6. Different topics could be combined in one question in the final exam.
7. Place a box around your final answer to each question.

Review topics

0: See Review 1 and Review 2.

1: Taylor series and operations with Taylor Series.

2: Taylor polynomials, approximation, error bounds, finding M, n, I etc.

3: Mass, moments and the center of Mass.

4: Double integrals, volume/area, double integrals in polar coordinates.

Practice problems for Taylor polynomials and Taylor Series

1. Consider the function

$$f(x) = \frac{x}{1-x^2} - \int_0^x e^{t^2}\,dt.$$

(a) Find the Taylor series for $f(x)$ based at $b = 0$. (b) Give the open interval of convergence for the Taylor series in part (a).

2. Consider the function

$$f(x) = 3\cos(2x) - \frac{\sin x}{x} + \frac{3}{1-x^2}.$$

(a) Find the Taylor series for $f(x)$ based at $b = 0$. (b) Find the first four nonzero terms in part (a). (c) Give the open interval of convergence for the Taylor series in part (a).

3. Consider the function

$$f(x) = x^2 \sin x$$

(a) Find the second Taylor polynomial $T_2(x)$ for $f(x)$ based at $b = 1$. (b) Use the Taylor inequality to bound the error on the interval $I = [0.8, 1.2]$.

Practice problems for double integrals

1. Find the center of mass of the lamina that occupies the region

$$D = \{(x, y) \quad : \quad x^2 + y^2 \leq 4\}$$

with density function $\rho(x, y) = 1 + (x^2 + 1)\sqrt{x^2 + y^2}$.

2. A lamina occupies the part of the disk $x^2 + y^2 \leq 1$ that lies in the first quadrant. Compute the mass of the lamina if the density function is $\rho(x, y) = xy + y^2 + 3$.

3. Consider the cardioid given by the polar function $r = 2 - \sqrt{2}(\cos(\theta) + \sin(\theta))$. Set up (but DO NOT EVALUATE) a double integral in polar coordinates that represents the area inside this cardioid and outside the circle centered at the origin with radius 2.

4. Compute

$$\int_0^1 \int_{\sqrt{y}}^1 \frac{1}{(x^3 + 1)} dxdy.$$

5. (Homework problem) Use a double integral to find the area of the region enclosed by one loop of the rose

$$r = 6\cos(3\theta).$$