Seattle Noncommutative Algebra Day

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ABSTRACT

The Brown-Goodearl Conjecture for weak Hopf algebras

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Brown and Goodearl conjectured that any Noetherian Hopf algebra should have finite injective dimension. The conjecture is known to be true in some cases, in particular for affine polynomial identity Hopf algebras. Weak Hopf algebras are an important generalization of Hopf algebras in which the axioms on the unit and counit are weakened. Just as for Hopf algebras, the category of modules over a weak Hopf algebra has a monoidal structure, and this has important consequences for homological properties of the algebra. We study the extension of the Brown-Goodearl conjecture to the case of weak Hopf algebras, and show that a weak Hopf algebra which is finite over an affine center has finite injective dimension, and is a direct sum of AS Gorenstein algebras. (Joint with Rob Won and James Zhang.)

Discriminants and hyperdeterminants

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The discriminant is an invariant associated to an algebra which is a finite rank free module over its centre. It is a classical invariant, dating back to Dedekind, which has been the star of some recent applications, such as the automorphism problem and Zariski cancellation problem. In positive characteristic, we may construct another invariant by taking

the hyperdeterminant of the hypermatrix of structure constants. We present a small result which suggests that these two invariants may be related in general.

Morphisms to Noncommutative projective lines

Adam Nyman

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Let k be a field, let C be a k-linear abelian category, let $\underline{\mathcal{L}} := \{\mathcal{L}_i\}_{i\in\mathbb{Z}}$ be a sequence of objects in C, and let $B_{\underline{\mathcal{L}}}$ be the associated orbit algebra. We describe sufficient conditions on $\underline{\mathcal{L}}$ such that there is a canonical morphism from the noncommutative space $\mathsf{Proj}B_{\underline{\mathcal{L}}}$ to the noncommutative projective line $\mathbb{P}^{nc}(\mathrm{Hom}(\mathcal{L}_0, \mathcal{L}_1))$, generalizing the usual construction of a map from a scheme X to $\mathbb{P}^1$ defined by an invertible sheaf $\mathcal{L}$ generated by two global sections.

We then discuss two applications: the construction of maps from noncommutative quadrics to $\mathbb{P}^1$ and, for every natural number $n > 1$, the construction of a noncommutative deformation, X_n, of an elliptic curve, together with a finite map from X_n to Piontkovski's nth noncommutative projective line.

This is joint work with Daniel Chan.

Algebraic structures in comodule categories over weak bialgebras

Robert Won

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A weak bialgebra is both an algebra and a coalgebra satisfying compatibility conditions which are weaker than those defining a bialgebra. These weakened compatibility conditions still ensure that the categories of modules and comodules over a weak bialgebra admit a monoidal structure. We will discuss algebra objects and coalgebra objects in comodule categories over weak bialgebras, and their relationships to

comodule algebras and comodule coalgebras. This work is joint with Chelsea Walton and Elizabeth Wicks.

Gelfand-Kirillov dimension of generalized Weyl algebras

Xiangui Zhao

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In this talk we will focus on the Gelfand-Kirillov dimension of generalized Weyl algebras (GWAs). Suppose $A = D[\sigma, a]$ is a GWA with base ring D. We prove that the difference of the Gelfand-Kirillov dimension of A and that of D could be any positive integer or infinity. Under mild conditions, this difference is exactly 1. As applications, we calculate the Gelfand-Kirillov dimensions of various algebras of interest, including the (quantized) Weyl algebras, ambiskew polynomial rings, noetherian (generalized) down-up algebras, iterated Ore extensions, quantum Heisenberg algebras, universal enveloping algebras of Lie algebras, quantum GWAs, etc.

In the case the base ring D is a polynomial ring of two variables, we prove that the difference of the Gelfand-Kirillov dimension of A and that of D is 1 if and only if the defining automorphism of A is conjugate to a triangular automorphism in the automorphism group of D.