The Finite Dual of Commutative-by-finite Hopf Algebras

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joint work with Ken Brown & Astrid Jahn

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k a field, $\bar{k} = k$, $\mathrm{char}\,(k) = 0$
$(H, m, u, \Delta, \epsilon, S)$ Hopf algebra over k

$H^* = \mathrm{Hom}(H, k)$
Finite dual:

$$H^\circ = \{f \in H^* : f(I) = 0, \text{ for some } I \triangleleft H \text{ of finite codimension}\}$$

Theorem (Kostant-Gabriel-Cartier)

Let H be an affine commutative Hopf algebra. Then, $H \cong \mathcal{O}(G)$ for some affine algebraic group G and

$$H^\circ \cong U(\mathfrak{g}) * kG,$$

where

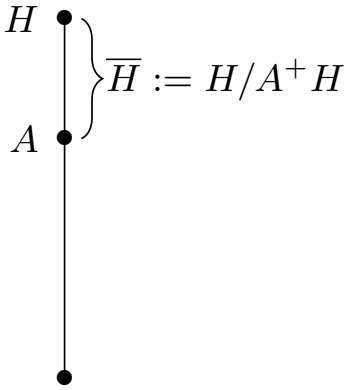
- $G \cong \mathrm{Alg}(H, k)$ *and* $\mathfrak{g} = \mathrm{Lie}\, G$.
- $U(\mathfrak{g}) = \{f \in H^\circ : f((H^+)^n) = 0, \text{ for some } n > 0\}$.
- G *acts on* $U(\mathfrak{g})$ *by conjugation.*

Affine commutative-by-finite Hopf algebras:

Affine k-Hopf algebras H that are left (or right) module-finite over some normal commutative Hopf subalgebra A.

A normal $\quad\Rightarrow\quad A^+H$ Hopf ideal

$\quad\Rightarrow\quad \overline{H} := H/A^+H$ Hopf algebra



Idea: decompose H° in terms of $\overline{H}^*$ and A°.

Theorem (B.-C.-J.)

Let H be an affine commutative-by-finite Hopf algebra. We write $H = A \oplus X$ as left (or right) A-modules.

1. *If X can be chosen to be a coideal of H, then as algebras*

$$H^\circ \cong \overline{H}^* * A^\circ.$$

2. *If X can be chosen to be an ideal of H, then as algebras*

$$H^\circ \cong \overline{H}^* *_\sigma A^\circ,$$

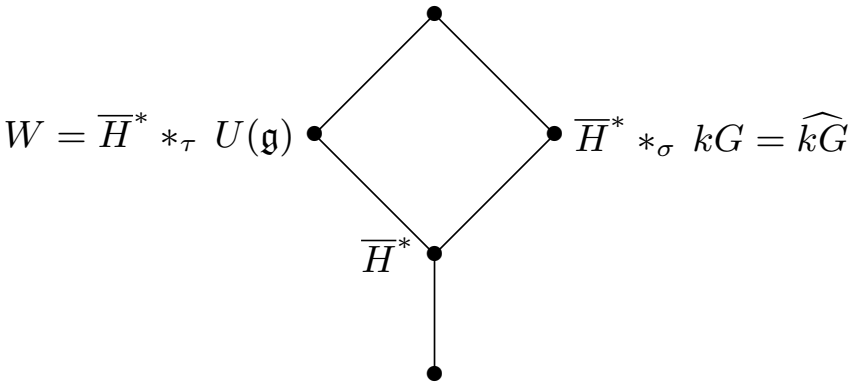
for some cocycle σ.

$$A^\circ \cong U(\mathfrak{g}) * kG,$$

- $U(\mathfrak{g}) = \{f \in A^\circ : f((A^+)^n) = 0, \text{ for some } n > 0\}$.
- $kG = \{f \in A^\circ : f(\mathbf{m}_{g_1} \cap \ldots \cap \mathbf{m}_{g_r}) = 0, \text{ for some } \mathbf{m}_{g_i} \lhd_{max} A\}$.

Under certain hypotheses,

$$H^\circ \cong \overline{H}^* *_\sigma (U(\mathfrak{g}) * kG)$$



References

- K.R. Goodearl & R.B. Warfield Jr., *An Introduction to Noncommutative Noetherian Rings*, London Mathematical Society, Student Texts **61**, 2004.
- K.R. Goodearl & J.J. Zhang, *Noetherian Hopf algebra domains of Gelfand-Kirillov dimension two*, Journal of Algebra 324, pp. 3131-3168, 2010.
- A. Jahn's Doctor of Philosophy thesis, *The finite dual of crossed products*, University of Glasgow, December 2014.
- S. Montgomery, *Hopf Algebras and Their Actions on Rings*, American Mathematical Society, 1992.
- D.E. Radford, *Hopf Algebras*, Series on Knots and Everything, Vol. 49, World Scientific Publishing, 2012.
- J.T. Stafford & J.J. Zhang, *Homological Properties of (Graded) Noetherian PI rings*, Journal of Algebra 168, pp. 988-1026, 1994.
- J. Wu, G. Liu & N. Ding, *Classification of affine prime regular Hopf algebras of GK-dimension one*, Advances in Mathematics 296, October 2014.

Thank you.