Math 126 **Final Examination** **Winter 2009**

Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

- This exam is closed book. You may use one $8\frac{1}{2} \times 11$ sheet of notes.
- Give your answers in exact form. Do not give decimal approximations.
- Graphing calculators are not allowed. Do not share notes.
- In order to receive credit, you must show your work. Do not do computations in your head. Instead, write them out on the exam paper.
- **Place a box around** $\boxed{\textbf{YOUR FINAL ANSWER}}$ **to each question.**
- If you need more room, use the backs of the pages and indicate to the reader that you have done so.
- Raise your hand if you have a question.

Problem	Total Points	Score
1	10	
2	10	
3	10	
4	14	
5	12	

Problem	Total Points	Score
6	10	
7	11	
8	11	
9	12	
Total	100	

1. [**10 points total**] Consider the three points $P = (1, 1, 1)$, $Q = (2, 0, 3)$ and $R = (-1, 3, 0)$.

(a) [**5 points**] Give the equation of the plane containing the points P, Q and R.

(b) [**5 points**] Find the coordinates of the point on the plane in part (a) that is closest to the point $(8, 7, -5)$.

2. [**10 points total**] Find all points (x, y) on the polar curve $r = 4\sin\theta$ where the tangent line is vertical.

3. [**10 points total**] The position function of a particle is given by $\mathbf{r}(t) = \left\langle 3\cos t, t^2 - t, 3\sin t \right\rangle$. (Here t is in seconds and x, y and z are measured in feet.) Compute the minimum speed of the particle.

4. [**14 points total**] Consider the function $F(x) = xe^{x-1}$.

(a) [**6 points**] Find the second Taylor polynomial T_2 of $F(x)$ based at $b = 1$.

(b) [**2 points**] Use the second Taylor polynomial T_2 to approximate $F(0.8)$.

(c) [**6 points**] Use Taylor's inequality to find an upper bound for the error in your approximation above.

5. [**12 points total**] Let $f(x) = \ln(e + 3x)$.

(a) [**6 points**] Find the Taylor series of the function $f(x)$ centered at $b = 0$.

(b) [**6 points**] Find an interval on which the series converges. Justify your answer.

6. [**10 points total**] Consider the helix given by $\mathbf{r}(t) = \langle \cos(t), \sin(t), 2t \rangle$.

(a) [**5 points**] Find the parametric equation of tangent line to the helix at the point $(0, 1, \pi)$.

(b) [**5 points**] Find the equation of the plane that contains the previous line and the point $(1, 1, 1)$.

7. [**11 points total**] Let $f(x, y) = \ln(y - x) \cdot \sqrt{25e^2 - (x - e)^2 - (y - e)^2}$.

(a) [**5 points**] Find and sketch the domain of f.

(b) [**6 points**] Consider the surface $z = f(x, y)$. Find the equation of the tangent plane to the surface at the point $x_0 = e$, $y_0 = 4e$.

8. [**11 points total**] Let $D = \{(x, y) \in \mathbb{R}^2 \mid 4 \leq x^2 + y^2 \leq 4x,\ y \geq 0\}$.

(a) [**5 points**] Draw a careful picture for the domain D.

(b) [**6 points**] Compute the area of D.

9. [**12 points total**] Locate and classify all critical points of the function $g(x, y) = 2x^2+y^3-6xy$.