

MATH 126 – Spring 2007  
Final Exam Hints, Answers, and Partial Solutions

1. (a) ANSWER:  $T_2(x) = 2 + 4(x - 1) + 3(x - 1)^2$   
(b) HINT: Taylor's inequality states that the error is bounded by  $\frac{M}{3!}|x - 1|^3$ , where  $M$  is an upper bound of  $f'''(x)$  on the interval  $J$ . But  $f'''(x) = 6$  for all values of  $x$ . So, we can take  $M$  to be 6 and we need an interval  $J$  on which  $|x - 1|^3 < 0.001$ .  
ANSWER:  $J = [0.9, 1.1]$

2. HINT: The Taylor series for  $\frac{e^{x^2} - 1}{x}$  is

$$x + \frac{x^3}{2!} + \frac{x^5}{3!} + \frac{x^7}{4!} + \dots$$

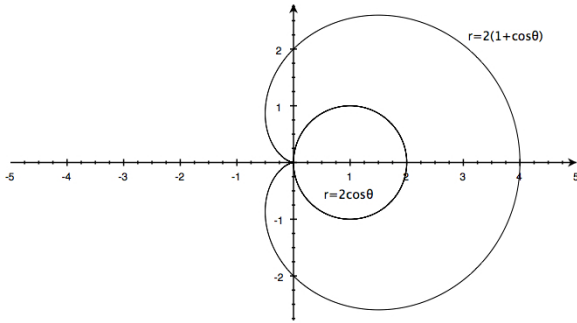
Further, since  $\frac{3}{(x-1)^2} = 3 \frac{d}{dx} \left( \frac{1}{1-x} \right)$ , the Taylor series for  $\frac{3}{(x-1)^2}$  is

$$3 + 6x + 9x^2 + 12x^3 \dots$$

ANSWER: The Taylor series for  $f(x)$  is

$$3 + 7x + 9x^2 + 12.5x^3 + \dots$$

3. (a) F; (b) T; (c) F; (d) T; (e) F; (f) T; (g) T; (h) F
4. (a) ANSWER:  $x = -1, y = t, z = 3 + t$   
(b) ANSWER:  $5x - 2y + 2z = 1$
5. ANSWER: 1
6. (a) ANSWER:  $t = \sqrt[6]{\frac{1}{2}}$   
(b) ANSWER:  $a_T = 0$
7. (a) ANSWER:  $y = \frac{3}{7}(x - 2) + 2$   
(b) ANSWER: The curve has a horizontal tangent at  $t = -\frac{1}{2}$  and vertical tangents at  $t = 0$  and  $t = -\frac{3}{4}$ .
8. (a) ANSWER: The domain of  $f$  is the set of points  $(x, y)$  such that  $y < 2x$  but  $y \neq 2x - 1$ .  
(b) ANSWER:  $z = -2e(x - e) + 5e(y - e) + 3e^2$   
(c) ANSWER:  $f(3, 3) \approx 9e$
9. ANSWER: The base of the glass side is 8.0505 meters and the height of the glass side is 4.0252 meters. The length of the pool is 30.85989 meters.
10. (a) ANSWER: The region  $R$  is the region between these two curves:



(b) ANSWER:  $5\pi$