Math 126 Final Examination Spring 2010

Print Your Name

Signature

Student ID Number

Quiz Section

Professor's Name

TA's Name

!!! READ...INSTRUCTIONS...READ !!!

1. Your exam contains 9 questions and 11 pages; PLEASE MAKE SURE YOU HAVE A COMPLETE EXAM.
2. The entire exam is worth 90 points. Point values for problems vary and these are clearly indicated. You have 2 hours and 50 minutes for this final exam.
3. Make sure to ALWAYS SHOW YOUR WORK; you will not receive any partial credit unless all work is clearly shown. If in doubt, ask for clarification.
4. There is plenty of space on the exam to do your work. If you need extra space, use the back pages of the exam and clearly indicate this.
5. You are allowed one 8.5×11 sheet of handwritten notes (both sides). Graphing calculators are NOT allowed; scientific calculators are allowed. Make sure your calculator is in radian mode.
6. Unless otherwise instructed, ALWAYS GIVE YOUR ANSWERS IN EXACT FORM. For example, 3π, $\sqrt{2}, \ln(2)$ are in exact form; the corresponding approximations $9.424778, 1.4142, 0.693147$ are NOT in exact form.

Problem	Total Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	

Problem	Total Points	Score
6	10	
7	10	
8	10	
9	10	
Total	90	

1. Let $f(x) = \dfrac{1}{1 + \frac{1}{2}x^2} - e^{(3x^2)}$.

   (a) Give the Taylor series for $f(x)$ based at $b = 0$. Write your answer using one sigma sign.

   (b) Give the open interval of convergence for the Taylor series in part (a).

   (c) Find the fourth Taylor polynomial, $T_4(x)$, for $f(x)$ based at $b = 0$.

2. The location of a particle is given by the vector function $\mathbf{r}(t) = \langle 3t-6, 2t^3-5t, -t^2+11 \rangle$.

(a) Find the **speed** of the particle at the instant when it passes through the yz-plane.

(b) Find all times when the tangential component of acceleration is zero.

3. Let $f(x) = x \ln x$.

(a) Find the second Taylor polynomial $T_2(x)$ for $f(x)$ based at the $b = 1$.

(b) Use the Quadratic Approximation Error Bound to find an interval J containing b so that the error bound is at most 0.01.

4. (a) Find the equation of the plane containing the line

$$x = 1 + 3t, y = 2 + 2t, z = 3 + t$$

and the point $(0, -2, 5)$.

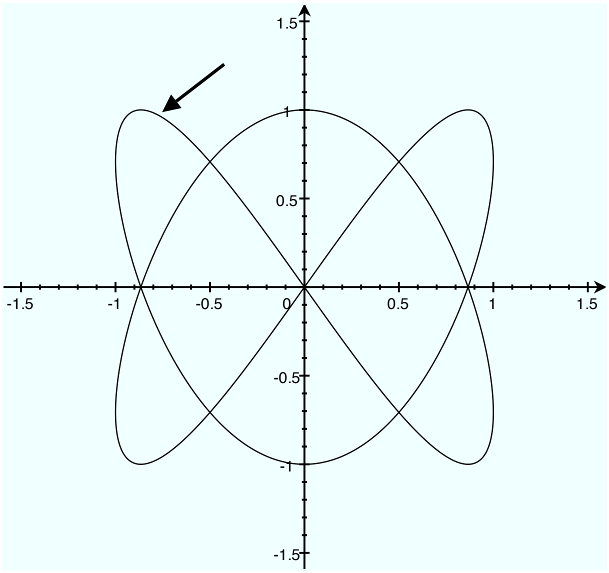
(b) Write the equation of the line of intersection of the two planes defined by:

$$2x - z = 0 \qquad \text{and} \qquad x + y + z = 1.$$

5. The curve

$$x = \sin 2t,\ y = \cos 3t$$

is shown below.



The point $t = 2$ is marked by the arrow.

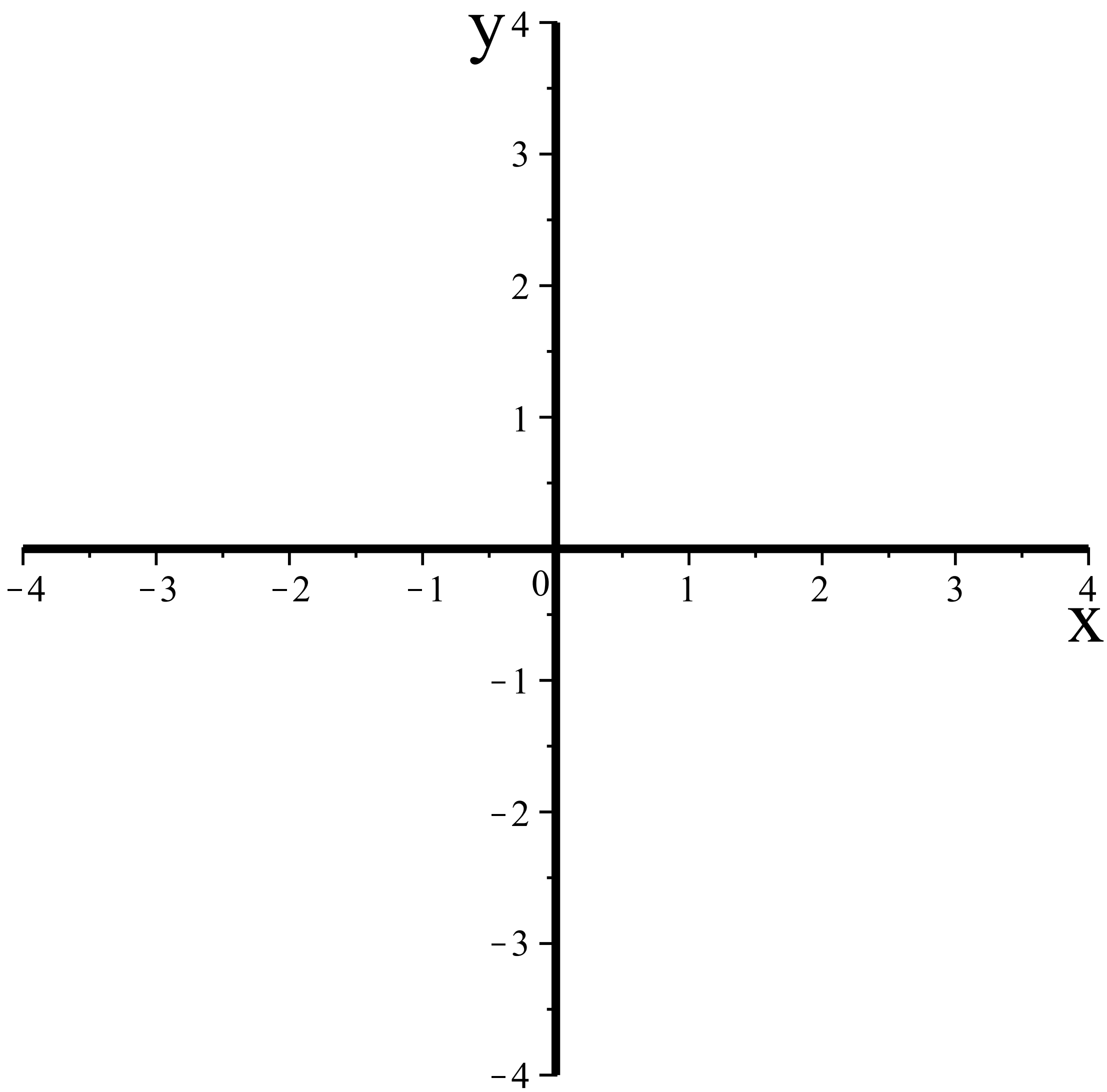
(a) Find the equation of the tangent line to the curve at the point $t = 2$.

(b) Find the curvature of the curve at the point $t = 2$.

6. Find the volume of the solid that lies under the plane $3x + 2y + z = 12$ and above the rectangle $R = [0, 1] \times [-2, 3]$.

7. Consider the function $f(x, y) = y - x^2$

a) Draw a contour diagram for f showing level curves for $z = -2, 0, 2$.



b) Find the equation of the tangent plane to the surface $z = y - x^2$ at the point $(1, 1, 0)$.

8. Consider the function $f(x, y) = x^2 + y^2 - xy$ over D, where D is the region enclosed by the circle of radius 4 centered at the origin.

   (a) Find and classify all critical points.

   (b) Find the absolute maximum value of $f(x, y)$ over D.

9. Consider the parametric curve $\mathbf{r}(t) = \langle \sin^2(t), \sin(t)\cos(t), \cos(t) \rangle$, for t between 0 and 2π.

   (a) Show that the tangent vector at every point is perpendicular to $\mathbf{r}(t)$.

   (b) Show that $\mathbf{r}(t)$ is always lying on the sphere of radius one.