

Your Name

Your Signature

Student ID #

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Quiz Section

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Professor's Name

TA's Name

- Turn off and put away all electronic devices except your non-graphing calculator.
- This exam is closed book. You may use one $8\frac{1}{2} \times 11$ sheet of handwritten notes (both sides may be used).
- Graphing calculators are not allowed. Do not share notes.
- In order to receive credit, you must show your work on the exam paper, with some explanation in English, if appropriate. Do not do computations in your head. Instead, write them out on the exam paper.
- Place a box around **YOUR FINAL ANSWER** to each question.
- If you need more room, use the **back of the previous page** and indicate to the reader that you have done so.
- Raise your hand if you have a question.

| Problem | Total Points | Score |
|---------|--------------|-------|
| 1 | 12 | |
| 2 | 12 | |
| 3 | 10 | |
| 4 | 8 | |
| 5 | 10 | |

| Problem | Total Points | Score |
|--------------|--------------|-------|
| 6 | 10 | |
| 7 | 10 | |
| 8 | 13 | |
| 9 | 15 | |
| Total | 100 | |

1. Let $f(x) = \frac{1}{5-x}$, $I = [-2, 2]$, and $b = 0$.

(a) [3 points] Find the first Taylor polynomial for $f(x)$ based at b .

(b) [4 points] Use Taylor's inequality to give a bound for the error $|f(x) - T_1(x)|$ on I .

(c) [5 points] Find an integer, n , so that the bound for the error $|f(x) - T_n(x)|$ on I given by Taylor's inequality is smaller than 0.05 and larger than 0.04.

2. (a) [4 points] Give the Taylor series for $f(x) = \frac{1}{2x^2 + 1} - \cos(3x)$ based at $b = 0$. Write your answer using one sigma sign.

(b) [4 points] Give the open interval of convergence of the Taylor series in part (a).

(c) [4 points] Find $T_4(x)$ in expanded notation. Simplify as much as possible.

3. (a) [2 points] Draw a picture of the region R bounded by the circles $x^2 + y^2 = 25$ and $x^2 + y^2 = 16$ in the first quadrant. Label at least two points.

- (b) [8 points] Evaluate the double integral

$$\int \int_R x + \sqrt{x^2 + y^2} \, dx dy.$$

4. [8 points] Consider the plane determined by the points $(-1, 2, 0)$, $(1, 2, 1)$, and $(0, 3, 4)$. Give parametric equations for the line that intersects the plane at $(0, 3, 4)$ and is perpendicular to the plane.

5. [10 points] Find and classify all the critical points of the function $f(x, y) = 3xy - x^3 - y^3$.

6. [10 points] Using the linear approximation at $(1, 1)$, estimate the value of the function $f(x, y) = \sin(xy - 1)$ at the point $(1.1, 1.2)$.

7. [10 points] Consider a curve given in polar coordinates by the equation $r = \sec(\theta + \pi/4)$. Find the equation of the tangent line (in Cartesian coordinates) at the point where $\theta = 0$. Simplify your answer as much as possible.

8. Consider a parametric curve given by $\vec{r}(t) = \langle \cos^2 t, \cos t \sin t, \sin t \rangle$, $0 \leq t < 2\pi$.

(a) [5 points] Show that the unit tangent vector $\vec{T}(t)$ is perpendicular to the position vector $\vec{r}(t)$.

(b) [5 points] Find the equation of the normal plane to the curve at the point $t = 0$.

(c) [3 points] Show that the origin belongs to the normal plane to the curve at any point t .

9. A constant force $\vec{F} = \langle 0, 10, 8 \rangle$ acts on a particle of mass $m = 2$. Newton's Law says that $\vec{F} = m\vec{a}$, where \vec{a} is the acceleration. At the time $t = 0$ the particle is located at the origin and has velocity $\vec{v} = \langle 1, -3, -2 \rangle$.

(a) [3 points] Determine the acceleration $\vec{a}(t)$ and the velocity $\vec{v}(t)$ of the particle at any time t .

(b) [3 points] Find the position of the particle at the time $t = 1$

(c) [6 points] Determine the tangential and normal components of the acceleration at the time $t = 1$.

(d) [3 points] Find the curvature at the point $t = 1$.