MATH 126 – Winter 2007
Final Exam Hints, Answers, and Partial Solutions

1. (a) ANSWER: $T_1(x) = \frac{1}{5} + \frac{1}{25}x$

   (b) HINT: Taylor's inequality states that the error is bounded by $\dfrac{M}{2}x^2$, where M is an upper bound for $|f''(x)|$. Here, $|f''(x)| = \dfrac{2}{|5-x|^3}$, which is largest on I when $x = 2$. So, we can take M to be $\frac{2}{27}$.
   ANSWER: error $\leq \frac{4}{27}$

   (c) HINT: Show that $f^{(k)}(x) = \dfrac{k!}{(5-x)^{k+1}}$ and that the error is bounded by $\dfrac{1}{3}\left(\dfrac{2}{3}\right)^{n+1}$. Then solve the system $0.04 < \dfrac{1}{3}\left(\dfrac{2}{3}\right)^{n+1} < 0.05$ for n.
   ANSWER: $n = 4$

2. (a) HINT: Substitute $-2x^2$ into the Taylor series for $\frac{1}{1-x}$ and $3x$ into the Taylor series for $\cos x$ and subtract.
   ANSWER: $\displaystyle\sum_{k=0}^{\infty}(-1)^k\left(2^k - \frac{3^{2k}}{(2k)!}\right)x^{2k}$

   (b) HINT: The Taylor series for $\cos x$ converges for all x, but the Taylor series for $\dfrac{1}{1-x}$ converges only for x such that $|x| < 1$. This means that the Taylor series for $f(x)$ converges only for x such that $|2x^2| < 1$.
   ANSWER: $-\dfrac{1}{\sqrt{2}} < x < \dfrac{1}{\sqrt{2}}$

   (c) ANSWER: $T_4(x) = \frac{5}{2}x^2 + \frac{5}{8}x^4$

3. (b) HINT: $\displaystyle\iint_R x + \sqrt{x^2+y^2}\, dA = \int_0^{\pi/2}\int_4^5 (r\cos\theta + r)r\, dr\, d\theta$
   ANSWER: $\dfrac{61}{3}\left(1 + \dfrac{\pi}{2}\right)$

4. ANSWER: $x = -t,\ y = 3 - 7t,\ z = 4 + 2t$

5. ANSWER: $(0,0)$ gives a saddle point; $(1,1)$ gives a local max

6. ANSWER: $f(1.1, 1.2) \approx 0.3$

7. ANSWER: $y = x - \sqrt{2}$

8. (a) HINT: Note that $\vec{T}(t)$ and $\vec{r}\,'(t)$ have the same direction. So showing that $\vec{r}(t)\cdot\vec{r}\,'(t) = 0$ is sufficient.

   (b) HINT: The vector $\vec{r}\,'(0)$ is a normal vector to the normal plane at the point $(1,0,0)$.
   ANSWER: $y + z = 0$

9. (a) ANSWER: $\vec{a}(t) = \langle 0, 5, 4\rangle$ and $\vec{v}(t) = \langle 1, 5t-3, 4t-2\rangle$.

   (b) ANSWER: $\vec{r}(1) = \langle 1, -\frac{1}{2}, 0\rangle$

   (c) ANSWER: $a_T = 6$, $a_N = \sqrt{5}$

   (d) ANSWER: $\kappa(1) = \frac{\sqrt{5}}{9}$