Math 126 Autumn 2015
Final Exam Answers

1. (a) Yes, the three points form a right triangle since $\overrightarrow{PQ} = \langle 1, 1, -3\rangle$ and $\overrightarrow{QR} = \langle 1, 2, 1\rangle$ and $\overrightarrow{PQ} \cdot \overrightarrow{QR} = 0$.
   (b) $7x - 4y + z = 2$
   (c) One possibility: $\mathbf{r}(t) = \langle 2\cos t, 2\sin t, 2 - 14\cos t + 8\sin t\rangle$
2. (a) One possibility: $x = 1 - 2t$, $y = -1 + 3t$, $z = 4t$
   (b) $-2x + 3y + 4z = 0$
3. (a) iii; (b) i; (c) ii and iv
4. $\left(\frac{1-t}{2}, \frac{1-t}{2}, \frac{|1-t|}{\sqrt{2}} + 1\right)$
5. $M_y = \frac{1}{2}e - 1$
6. (a) $t = -1$, $t = 1$
   (b) No. The binormal vector is parallel to $\mathbf{r}'(t) \times \mathbf{r}''(t) = \langle -3\sin t, -2\cos t, -6\rangle$. This is parallel to the z-axis if and only if it is a constant multiple of $\langle 0, 0, 1\rangle$. This would happen only at a value of t such that $\sin t = \cos t = 0$ and there is no such t.
   (c) $a_T = \dfrac{-5\sin t\cos t}{\sqrt{4\sin^2 t + 9\cos^2 t + 1}}$
   (d) infinitely many (those values of t such that $\sin t$ or $\cos t$ is 0)
7. (a) (A)
   (b) $\frac{3\pi}{4} - 2$
   (c) $y = 2$ and $y = -\frac{1}{4}$
8. $f(x) = x^2 + x^3 - x^4 + \frac{3}{2}x^6 + \ldots$
9. (a) $T_2(x) = x + x^2$
   (b) One possibility: $|f(x) - T_2(x)| < \frac{1}{6}$
   (Uses the bound $|f'''(x)| = 2e^x|\cos x - \sin x| < 2e^x(2) \leq 4e^{1/2} < 4\sqrt{4} = 8$ on $\left[-\frac{1}{2}, \frac{1}{2}\right]$.)