Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

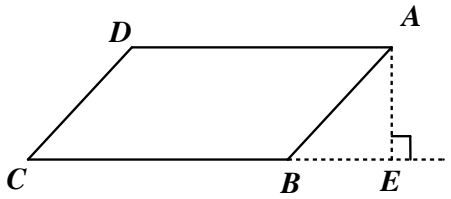
TA's Name

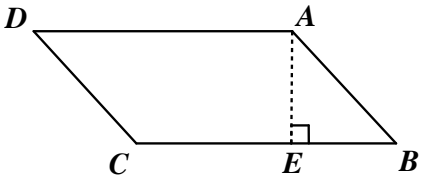
- CHECK that your exam contains 9 problems on 10 pages.
- This exam is closed book. You may use one $8\frac{1}{2} \times 11$, double-sided, hand-written sheet of notes and a TI-30X IIS calculator. Do not share notes or calculators.
- Unless otherwise specified, you should give your answers in exact form. (For example, $\frac{\pi}{4}$ and $\sqrt{2}$ are in exact form and are preferable to their decimal approximations.)
- In order to receive full credit, you must show all of your work.
- **Place a box around YOUR FINAL ANSWER to each question.**
- If you need more room, use the backs of the pages and indicate to the reader that you have done so. DO NOT USE SCRATCH PAPER.
- Raise your hand if you have a question.

Problem	Total Points	Score
1	14	
2	10	
3	10	
4	12	
5	10	

Problem	Total Points	Score
6	12	
7	10	
8	10	
9	12	
Total	100	

1. (14 points) A parallelogram, which is not a rectangle, has three of its consecutive vertices given by $A(1, 2, 4)$, $B(2, 0, 5)$ and $C(4, 6, 0)$. The line through the points A and E intersects the line through B and C at a right angle. Depending on whether point E falls between B and C or not, there are two pictures possible, as shown below.





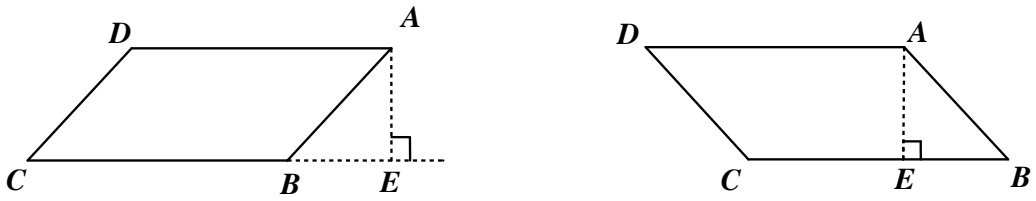
(a) Find the coordinates of the fourth vertex D of the parallelogram.

(b) Find the area of the parallelogram.

(This problem continues on the next page.)

(This problem is continued from the previous page.)

Again, the two possible pictures are:



(c) Find the coordinates of the point E and decide which of the two pictures is a correct representation: the picture on the LEFT or the picture on the RIGHT?

2. (10 point) Given the points $P(2, 0, 5)$, $Q(-3, 2, 0)$, $R(2, 1, 4)$ and $S(0, 5, 0)$:

   (a) Find the equation of the plane containing the three points P, Q, and R.

   (b) Find the parametric equations of the line through S that is perpendicular to the plane you found above.

   (c) Find the coordinates of the point where the line in part (b) intersects the plane in part (a).

3. (10 points) The acceleration vector of a particle at time t is given by the formula

$$\mathbf{a}(t) = \cos(2t)\mathbf{i} + \sin(2t)\mathbf{j} + \mathbf{k}.$$

(a) Let $\mathbf{v}(t) = v_1(t)\mathbf{i} + v_2(t)\mathbf{j} + v_3(t)\mathbf{k}$ denote the velocity of the particle at time t, and suppose that the velocity is zero at time $t = 0$. Find $v_1(t)$, $v_2(t)$, and $v_3(t)$.

(b) Now let $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$ denote the position of the particle at time t and suppose that the particle is at the origin at time $t = 0$. Using the result of part (a), find $x(t)$, $y(t)$, and $z(t)$.

4. (12 points) Consider the curve given by $\mathbf{r}(t) = \langle e^{-2t}, \tan(t), t^3 + t \rangle, -\pi/2 < t < \pi/2$.

   (a) Find $\mathbf{r}'(t)$.

   (b) Find the unit tangent vector to the curve at the point given by $\mathbf{r}(0)$.

   (c) Find parametric equations for the tangent line to the curve at the point given by $\mathbf{r}(0)$.

   (d) Is the point $(1, 1, 1)$ on the tangent line you found in part (c)? Justify your answer.

5. (10 points) Consider the curve given in polar form by $r = e^{(3\theta)}$.

   (a) What is the slope of the tangent line to the curve when $\theta = \pi/2$. Be sure to simplify your answer!

   (b) Where does the tangent line in part (a) intersect the x-axis? Give your answer in exact form (no decimals). Simplify your answer as much as possible.

6. (12 points) Find the absolute maximum and absolute minimum values of the function

$$f(x, y) = x^2 + y^2 - x^2y + 7$$

on the set $D = \{(x, y) : |x| \leq 1, |y| \leq 1\}$.

7. (10 points) Find the positive constant K such that the solid bounded by the xy-plane and the paraboloid $z = K - x^2 - y^2$ has volume 4000. Give an exact answer.

8. (10 points) Consider the function $f(x) = x^2 \sin(x^4)$.

   (a) Find the Taylor series for $f(x)$, based at $b = 0$. Write your answer using $\sum$ notation.

   (b) Find $T_3(x)$, the third Taylor polynomial for $f(x)$ based at $b = 0$.

   (c) What is the smallest value of $|f(x) - T_3(x)|$ on the interval from -1 to $\pi/2016$?

9. (12 points) Let $F(x)$ be the function $\int_0^x e^{t^2} dt$. Let $G(x)$ be the function $\int_0^x F(t)dt$. Let $H(x)$ be the function $\int_0^x G(t)dt$.

   (a) Find the Taylor series for $F(x)$, based at $b = 0$. Write your answer using $\sum$ notation

   (b) Find $T_3(x)$, the third Taylor polynomial of the function $G(x)$ based at $b = 0$.

   (c) Use Taylor's Inequality to find an upper bound for the error $|H(x) - T_2(x)|$ on the interval $[0, 4]$, where $T_2(x)$ is the second Taylor polynomial of $H(x)$, based at $b = 0$.