Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

- CHECK that your exam contains 8 problems on 6 double-sided pages, including this cover sheet. The back of the first page and both sides of the last page are reserved for scratch-work.
- This exam is closed book. You may use one $8\frac{1}{2}$" $\times$ 11" sheet of notes and a TI-30X IIS calculator. Do not share notes or calculators.
- Unless otherwise specified, you should give your answers in exact form. (For example, $\frac{\pi}{4}$ and $\sqrt{2}$ are in exact form and are preferable to their decimal approximations.)
- In order to receive full credit, you must show all of your work.
- **Place a box around YOUR FINAL ANSWER to each question.**
- If you need more room, use the back of the first page or either side of the last page **and indicate that you have done so**. If you *still* need more room, ask for more scratch paper.
- **Do not write within 1 centimeter of the edge of the page.**
- Raise your hand if you have a question.

Problem	Total Points	Score
1	15	
2	12	
3	14	
4	8	
5	14	

Problem	Total Points	Score
6	12	
7	15	
8	10	
Total	100	

You may use this page for scratch-work.

All work on this page will be ignored unless you write & circle "see first page" below a problem.

1. *(3 points per part)* Suppose $\mathbf{a}$ and $\mathbf{b}$ are nonzero vectors in $\mathbf{R}^3$. Decide whether each of the following statements is **always** true, **sometimes** true, or **never** true. (Circle one.)

   If your answer is **always** or **never**, briefly explain why (one sentence is enough).

   If your answer is **sometimes**, give an example where it's true **and** an example where it's false.

   (a) $\mathbf{a} \cdot \mathbf{a} > 0$ **Always** **Sometimes** **Never**

   Remember, for full credit, you must include a short explanation (for **Always** *or* **Never***) or examples (for* **Sometimes***)!*

   (b) $\mathbf{a} \times \mathbf{b} = 2\mathbf{a}$ **Always** **Sometimes** **Never**

   (c) $|\mathbf{a} \times \mathbf{b}| = \mathbf{a} \cdot \mathbf{b}$ **Always** **Sometimes** **Never**

   (d) $\text{comp}_{\mathbf{a}}\mathbf{b} > |\mathbf{b}|$ **Always** **Sometimes** **Never**

   (e) $\text{proj}_{\mathbf{a}}\mathbf{b} = \mathbf{b}$ **Always** **Sometimes** **Never**

2. *(4 points per part)* Consider the vector function $\mathbf{r}(t) = \langle 3\cos(t) + 1, 4\cos(t) + 2, 5\sin(t) + 7\rangle$.

   (a) The space curve for $\mathbf{r}(t)$ lies in a plane. Find the equation of that plane.

   (b) Find parametric equations for the line tangent to $\mathbf{r}(t)$ at $(1, 2, 2)$.

   (c) Find $\mathbf{T}(t)$, the unit tangent vector to $\mathbf{r}(t)$.

3. *(7 points per part)* Consider the function $f(x, y) = xy - xy^3$.

   (a) Find all the critical points of f on $\mathbf{R}^2$ and classify each critical point.

   (b) Find the absolute maximum and minimum values of f on the triangular region bounded by the lines $y = x$, $y = 1$ and $x = 0$.

4. *(8 points)* Find $\dfrac{\partial z}{\partial x}$ if x, y, z are related by the implicit equation

$$x \sin z + e^{xy} = z.$$

5. *(7 points per part)* Compute the following integrals.

   (a) $\displaystyle\int_0^1 \int_0^{\cos^{-1}(y)} \sin(\sin(x))\,dx\,dy.$

   (b) $\displaystyle\int_0^1 \int_x^{\sqrt{2-x^2}} e^{x^2+y^2}\,dy\,dx.$

6. *(12 points)* A lamina occupies the rectangle $\mathcal{R} = [0, 4] \times [0, 2]$. Find its center of mass if the density at each point is given by the function $\rho(x, y) = x + y^2$.

7. *(5 points per part)* For all parts, consider $f(x) = \ln(x + 2)$ based at $b = 1$. (**NOT based at zero!**)

   (a) Find the third Taylor polynomial, $T_3(x)$, for $f(x)$ based at $b = 1$.

   (b) Use Taylor's inequality to find an upper bound (as sharp as possible) for the error $|f(x) - T_2(x)|$ on the interval $[-0.5, 2.5]$, where $T_2(x)$ is the second Taylor polynomial of $f(x)$ centered at $b = 1$.

   (c) Find the smallest value of n such that Taylor's inequality guarantees that the error $|f(x) - T_n(x)| < 0.02$ for all x in the interval $[-0.5, 2.5]$, where $T_n(x)$ is the n^{th} Taylor polynomial of $f(x)$ centered at $b = 1$.

8. Consider the function $f(x) = x\sin(x^2)$.

   (a) *(6 points)* Find the Taylor series of $f(x) = x\sin(x^2)$ based at $b = 0$. Use the sigma sum notation $\sum_{k=\dots}^{\infty}$ to express the Taylor series.

   (b) *(4 points)* Use the series found in (a) to find $f^{(507)}(0)$ (i.e., the 507^{th} order derivative of f at 0.)

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