Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

- This exam contains 9 problems. CHECK THAT YOU HAVE A COMPLETE EXAM.
- This exam is closed book. You may use one $8\frac{1}{2} \times 11$ sheet of notes and a non-graphing, scientific calculator. Do not share notes or calculators.
- Give your answers in exact form. Do not give decimal approximations.
- In order to receive credit, you must show your work.
- **Place a box around YOUR FINAL ANSWER to each question.**
- If you need more room, use the backs of the pages and indicate to the reader that you have done so.
- Raise your hand if you have a question.

Problem	Total Points	Score
1	10	
2	12	
3	12	
4	10	
5	12	

Problem	Total Points	Score
6	10	
7	10	
8	12	
9	12	
Total	100	

1. (10 points) Find the equation for the plane containing the point $(1, 2, 0)$ and the line given by the parametric equations $x = 5 + t$, $y = 3 - t$, $z = 2t$.

2. (12 points) Consider the surface $z = x^2 - 3y^2$.

   (a) Describe the traces parallel to the given plane. That is, state whether the traces are parabolas, circles, ellipses, hyperbolas, etc. You do not need to justify your answers.

      i. The traces parallel to the yz-plane (when x is fixed) are:

      ii. The traces parallel to the xz-plane (when y is fixed) are:

      iii. The traces parallel to the xy-plane (when z is fixed) are:

   (b) Name the surface given by $z = x^2 - 3y^2$. That is, state whether this equation gives a cone, an ellipsoid, a parabolic cylinder, etc. Again, no work is necessary. Just give your answer.

   (c) Find all (x, y, z) intersection points of the line through $(0, 0, 10)$ and $(2, 1, 7)$ with the surface $z = x^2 - 3y^2$.

3. (12 points) A particle's location at time t is given by the position function

$$\mathbf{r}(t) = \langle \cos(\pi t), \sin(\pi t), t^3 \rangle.$$

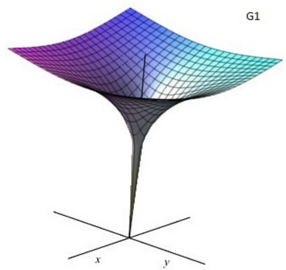
(a) Find the tangential component of acceleration at $t = 1$.

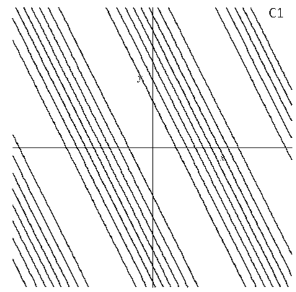
(b) Find the minimum speed.

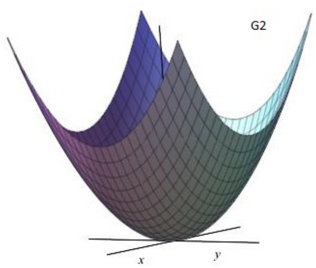
(c) The curve $\mathbf{h}(s) = \langle s - 2, s^2 - 9, 8 \rangle$ and the curve $\mathbf{r}(t)$ have one point of intersection. Find the angle between the curves at the intersection point, to the nearest degree.

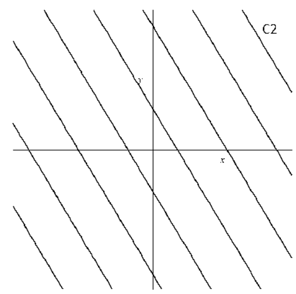
4. (10 points) Fill the table below matching the equations with their contour maps and graphs.

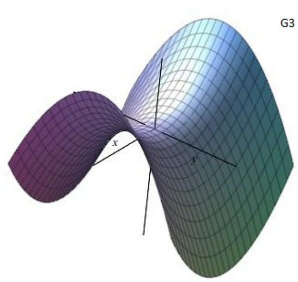
Function	$z = 5x + 3y$	$z = (x^2 + y^2)^{0.1}$	$z = \cos(4x + 2y)$	$z = x^2 + y^2$	$z = 2x^2 - 3y^2$
Graph (enter G1–G5)					
Contour map (enter C1–C5)					

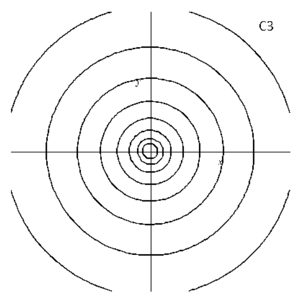


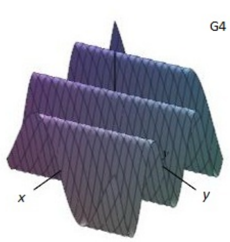


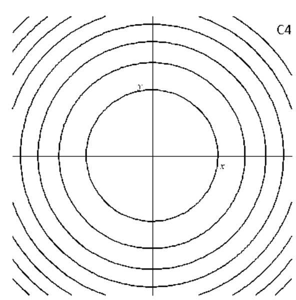


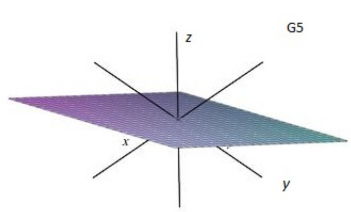


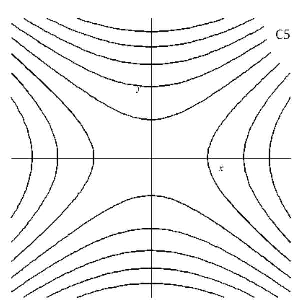












5. (12 points) The following questions regard the hyperboloid of one sheet $2x^2 + 3y^2 - z^2 = 1$ and the point $Q(3, 4, 0)$.

   (a) Find the points P and R on the hyperboloid that are closest to the point Q. P has the positive z coordinate. Remember to use the second derivatives test to verify that your points are the closest points.

   (b) Find the equation of the tangent plane to the hyperboloid at the point P.

   (c) Verify that the plane you found in part (b) is orthogonal to the line through the points P and Q.

6. (10 points) D is a region in the xy-plane and for a continuous function $f(x, y)$,

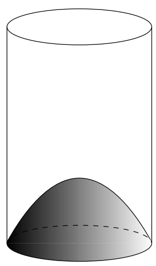
$$\iint_D f(x,y)\,dA = \int_0^6 \int_{2-\frac{x}{3}}^{2+\frac{x}{4}} f(x,y)\,dy\,dx + \int_6^8 \int_{(x-6)^2}^{2+\frac{x}{4}} f(x,y)\,dy\,dx.$$

Sketch the region D and reverse the order of integration.

7. (10 points) The picture below shows a right circular cylinder bounded by the planes $z = 0$, $z = 20$, and the surface $x^2 + y^2 = 20$. Inside the cylinder, you see a portion of the paraboloid

$$z = C\left(1 - \frac{x^2}{20} - \frac{y^2}{20}\right).$$

Find the value of C so that the volume inside the cylinder and above the paraboloid is the same as the volume inside the cylinder and below the paraboloid.



8. (12 points)

(a) Find the 2nd Taylor polynomial, $T_2(x)$, for $f(x) = e^x \sin x$ based at $b = 0$.

(b) Use the Quadratic Approximation Error Bound to bound the error $|f(x) - T_2(x)|$ on the interval $I = [-0.2, 0.2]$.

(c) Find a number a, with $0 < a < 1$, so that the error $|f(x) - T_2(x)|$ on the interval $J = [-a, a]$ is at most 0.01.

9. (12 points) Let $g(x) = x^4 e^{5x^2}$.

   (a) Find the Taylor series for $g(x)$ based at 0. Write your answer in summation notation using one Σ and give the first four non-zero terms of the Taylor series for $g(x)$ based at 0.

   (b) Use the Taylor series from part (a) to compute $g^{(100)}(0)$.

   (c) Let $T_6(x)$ be the 6th Taylor polynomial for $g(x)$ based at 0. Approximate $\displaystyle\int_{-0.25}^{0.25} g(x)\,dx$ by calculating $\displaystyle\int_{-0.25}^{0.25} T_6(x)\,dx$.