FINAL EXAM ANSWERS
MATH 126 SPRING 2012

1. $2x - 8y - 5z + 14 = 0$

2. (a) i. parabolas
   ii. parabolas
   iii. hyperbolas
   (b) hyperbolic paraboloid
   (c) $(4, 2, 4)$ and $(-10, -5, 25)$

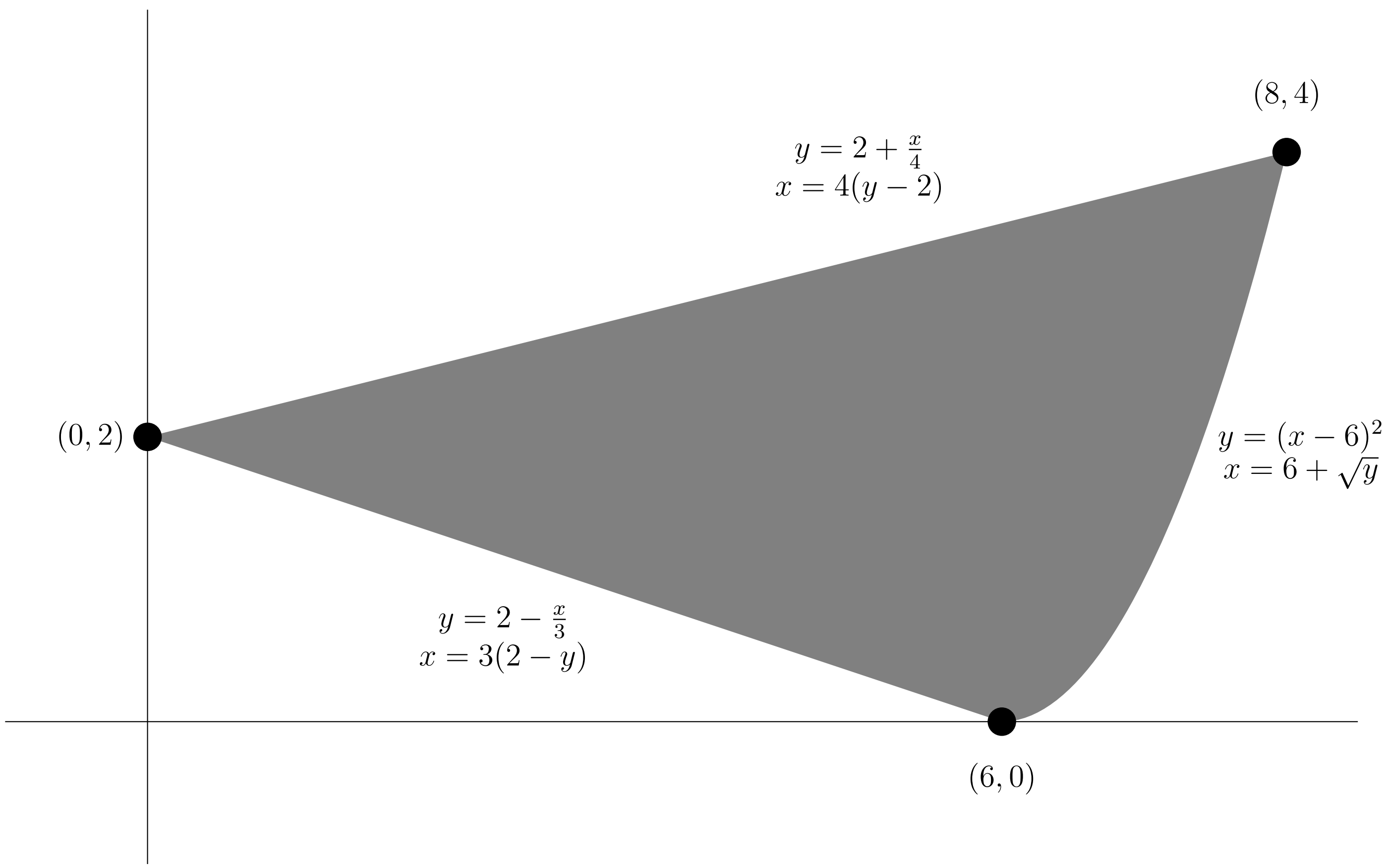
3. (a) $a_T = \frac{18}{\sqrt{\pi^2+9}}$
   (b) π
   (c) $\theta = \cos^{-1}\left(\frac{6\pi}{\sqrt{\pi^2+144}\sqrt{37}}\right) \approx 76°$

4.

Function	$z = 5x + 3y$	$z = (x^2 + y^2)^{0.1}$	$z = \cos(4x + 2y)$	$z = x^2 + y^2$	$z = 2x^2 - 3y^2$
Graph	G5	G1	G4	G2	G3
Contour Map	C2	C3	C1	C4	C5

5. (a) $P(1, 1, 2)$ and $R(1, 1, -2)$
   (b) $2x + 3y - 2z = 1$

6. .



$$\iint_D f(x,y)\,dA = \int_0^2 \int_{3(2-y)}^{6+\sqrt{y}} f(x,y)\,dx\,dy + \int_2^4 \int_{4(y-2)}^{6+\sqrt{y}} f(x,y)\,dx\,dy$$

7. HINT: The volume of the cylinder is $\pi r^2 h$, where $r = \sqrt{20}$ and $h = 20$. So, the volume of the cylinder is 400π. The volume inside the cylinder and below the paraboloid is given by

$$V = \iint_D C\left(1 - \frac{1}{20}(x^2 + y^2)\right) dA.$$

Convert to polar coordinates to compute this integral and find the value of C so that $V = \frac{1}{2}(400\pi)$.

ANSWER: $C = 20$

8. (a) $T_2(x) = x + x^2$

(b) HINT: $f'''(x) = 2e^x(\cos x - \sin x)$. For all values of x, $|\cos x - \sin x| \leq 2$. If x is in the interval I and t is between 0 and x, then $|f'''(t)| \leq 4e^t \leq 4e^{0.2} \leq 4e$.

ANSWER: $|f(x) - T_2(x)| \leq \frac{4e}{3!}(0.2)^3 = 0.014497503...$ (Other answers are possible.)

(c) HINT: Since $a < 1$, if x is in $J = [-a, a]$ and t is between 0 and x, then $|f'''(t)| \leq 4e$. So, $|f(x) - T_2(x)| \leq \frac{4e}{6}a^3$.

ANSWER: $a = \left(\frac{0.06}{4e}\right)^{1/3} = 0.176711....$ (Other answers are possible.)

9. (a) $g(x) = \sum_{k=0}^{\infty} \frac{5^k x^{2k+4}}{k!} = x^4 + 5x^6 + \frac{25}{2}x^8 + \frac{125}{6}x^{10} + ...$

(b) $g^{(100)}(0) = \frac{5^{48}100!}{48!}$

(c) $\int_{-0.25}^{0.25} g(x)\,dx \approx \int_{-0.25}^{0.25} T_6(x)\,dx = \int_{-0.25}^{0.25} x^4 + 5x^6\,dx = ... = 0.000477818$