MATH 126 FINAL EXAM ANSWERS
WINTER 2014

1. (a) T; (b) F; (c) F; (d) F; (e) F; (f) F; (g) F; (h) F.

2. (a) $5\sqrt{2}$
   (b) $\left(\frac{9}{2}, \frac{3}{2}, \frac{9}{2}\right)$

3. (a) $2x - y + z = 0$
   (b) $x = 2 + t, y = 4 + 4t, z = 2t$

4. (a) $v(t) = 13$; $a_T = 0$; $a_N = 13$
   (b) $\kappa(t) = \frac{1}{13}$
   (c) $x = 1 + t, y = -12, z = 6$

5. (a) f has saddle points at $\left(0, \frac{\pi}{2}\right)$ and $\left(\pi, \frac{\pi}{2}\right)$, a local maximum at $\left(\frac{\pi}{2}, 0\right)$, and a local minimum at $\left(\frac{\pi}{2}, \pi\right)$
   (b) The absolute maximum value of f on D is $f\left(\frac{\pi}{2}, 0\right) = 1$ and the absolute minimum is $f\left(x, \frac{\pi}{2}\right) = 0$, where x is any value in the interval $0 \leq x \leq \frac{\pi}{2}$.

6. (a) $f_x(x, y) = yx^{y-1} + \ln(y)y^x,\ f_y(x, y) = \ln(x)x^y + xy^{x-1}$
   (b) $z = x + y$
   (c) $f(1.01, 0.99) \approx 2$

7. (a) $z = -x$
   (b) the line $y = -x$

8. $$\left(0, \frac{15\pi + 30 + (10e^4 - 2e^2)\sqrt{2}}{\frac{56}{3}\sqrt{2} + (3e^4 - e^2)\frac{\pi}{2}}\right)$$

9. (a) HINT: Note that $f(x) = \ln(x^2 + 3x) = \ln(x) + \ln(x + 3)$.
   Taylor series for $f(x)$: $\ln(4) + \displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}\left(1 + \frac{1}{4^k}\right)}{k}(x - 1)^k$
   (b) Taylor series for $F(x)$: $\ln(4)(x - 1) + \displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}\left(1 + \frac{1}{4^k}\right)}{k(k + 1)}(x - 1)^{k+1}$
   (c)
   $$T_5(x) = \ln(4)(x-1) + \frac{\left(1 + \frac{1}{4}\right)}{2}(x-1)^2 - \frac{\left(1 + \frac{1}{4^2}\right)}{6}(x-1)^3 + \frac{\left(1 + \frac{1}{4^3}\right)}{12}(x-1)^4 - \frac{\left(1 + \frac{1}{4^4}\right)}{20}(x-1)^5$$

10. (a) $T_2(x) = 1 + x + \frac{3}{2}x^2$
    (b) $|f(x) - T_2(x)| \leq \dfrac{185e^3}{6}$ (other reasonable answers accepted)
    (c) 0