Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

- CHECK that your exam contains 8 problems.
- This exam is closed book. You may use one $8\frac{1}{2} \times 11$ sheet of notes and a scientific calculator with no graphing, programming, or calculus capabilities. Do not share notes or calculators.
- Unless otherwise specified, you should give your answers in exact form. (For example, $\frac{\pi}{4}$ and $\sqrt{2}$ are in exact form and are preferable to their decimal approximations.)
- In order to receive full credit, you must show all of your work.
- **Place a box around YOUR FINAL ANSWER to each question.**
- If you need more room, use the backs of the pages and indicate to the reader that you have done so. DO NOT USE SCRATCH PAPER.
- Raise your hand if you have a question.

Problem	Total Points	Score
1	12	
2	12	
3	13	
4	14	

Problem	Total Points	Score
5	12	
6	12	
7	12	
8	13	
Total	100	

1. (12 points) Find an equation for the plane containing the point $(1, 1, 1)$ and the line

$$(x, y, z) = (1 + 2t, 3 - t, t).$$

2. (12 points) Consider surfaces of the form $x^2 + by^2 + cz^2 = 0$, where b and c real numbers.

   (a) Describe the horizontal traces (slices by planes parallel to the xy-plane) of these surfaces:

      i. when $b > 0$ and $c < 0$.

      ii. when $b < 0$ is negative and $c > 0$.

   (b) Describe all pairs of numbers (b, c) such that the surface $x^2 + by^2 + cz^2 = 0$ contains the line $(x, y, z) = (4t, 2t, t)$. Your answer should include an equation that relates b and c.

   (c) Consider the horizontal trace $z = 1$ in the surfaces of part (b) (i.e., those that contain the given line). For which values of b and c is the horizontal trace a circle?

3. (13 points) On a very windy day, a math student launches a small water balloon into the air (towards an instructor). At time $t = 0$, the balloon is at the origin. The **velocity vector** for the balloon at time t seconds is given by $\mathbf{r}'(t) = \langle 6 + 2t, 4, 15 - 10t \rangle$.

   (a) Find the speed of the balloon at the positive time, t, when it hits the xy-plane.

   (b) Find the time at which the tangential component of acceleration for the balloon is zero.

   (c) Determine if each of the following is true or false **at the time you found in part (b)**. No work is needed to answer this question. Circle your answer.

   T **F** The acceleration $\mathbf{a}$ is orthogonal to the unit tangent vector $\mathbf{T}$.

   T **F** The acceleration $\mathbf{a}$ is orthogonal to the principal unit normal vector $\mathbf{N}$.

   T **F** The acceleration $\mathbf{a}$ is orthogonal to the binormal vector $\mathbf{B}$.

4. (14 points) Consider the curve $\mathbf{r}_1(t) = \langle t, 4-t, 25-t^2 \rangle$.

   (a) Find the point (x, y, z) where the tangent line to the curve $\mathbf{r}_1(t)$ at $t = 2$ intersects the plane $2x - 3y + z = 0$.

   (b) A second curve is given by $\mathbf{r}_2(s) = \langle 4, \sin(5s), 9-3s \rangle$. The two curves $\mathbf{r}_1(t)$ and $\mathbf{r}_2(s)$ have one point of intersection. Find the angle of intersection θ correct to the degree.

5. (12 points) The equation $2x + y - z = 2$ defines a plane in $\mathbb{R}^3$. Find all points on that plane at which the following function has a local minimum:

$$f(x, y, z) = x^2 + xz + 2y^2 - 2xy.$$

Use the Second Derivative Test to show that the point you found does indeed give a local minimum.

6. (12 points) Compute the integral

$$\int_0^1 \int_{\sqrt[3]{y}}^1 y e^{-2x^7} \, dx \, dy$$

7. (12 points) Let $A(x) = \int_0^x e^{-t^2} dt$.

   (a) Find the first Taylor polynomial (linear approximation) $T_1(x)$ and the second Taylor polynomial $T_2(x)$ for $A(x)$ based at $b = 0$.

   (b) Use the $T_2(x)$ you found to approximate $A\left(\frac{1}{2}\right) = \int_0^{1/2} e^{-t^2} dt$.

   (c) Find an upper bound on the error $\left|T_2\left(\frac{1}{2}\right) - A\left(\frac{1}{2}\right)\right|$.

8. (13 points) Let $f(x) = x \arctan x - \frac{1}{2} \ln (1 + x^2)$

   (a) Find the Taylor series for $f(x)$ based at $b = 0$. Express your answer using one sigma sign (Σ) and write out the first three non-zero terms of the series.

   (b) Find the open interval where the Taylor series in (a) converges.