Math 126 Winter 2015
Final Exam Answers

1. $x - 2y - 4z = -5$

2. (a) i. If $z = 0$, then the trace is $x^2 + by^2 = 0$, which is a single point $(0,0)$. Otherwise, since $z^2 > 0$, the traces have the form $x^2 + by^2 = d$, with $b, d > 0$. These are all ellipses.

   ii. If $z = 0$, then the trace is $x^2 + by^2 = 0$, which is a pair of lines. Otherwise, since $z^2 > 0$, the traces have the form $x^2 + by^2 = d$, where $b < 0$ and $d > 0$. These are all hyperbolas that do not intersect the x-axis.

   (b) In order to contain the line, the equation $16t^2 + 4bt^2 + ct^2 = 0$ must hold for all t, which means that $16 + 4b + c = 0$.

   (c) The trace in question has equation $x^2 + by^2 = 16 + 4b$. This is a circle precisely when $b = 1$.

3. (a) The balloon hits the xy-plane at $t = 3$. Speed at $t = 3$ is $\sqrt{385}$.

   (b) $t = \dfrac{138}{104}$

   (c) T; F; T

4. (a) $(-17, 21, 97)$

   (b) 66°

5. $\left(\dfrac{8}{23}, \dfrac{2}{23}, -\dfrac{28}{23}\right)$

6. $-\dfrac{1}{28}(e^{-2} - 1)$

7. (a) $T_1(x) = T_2(x) = x$

   (b) $A\left(\dfrac{1}{2}\right) \approx T_2\left(\dfrac{1}{2}\right) = \dfrac{1}{2}$

   (c) One possible answer: error $\leq \frac{1}{24}$

8. (a) $f(x) = \displaystyle\sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+2}}{(2k+1)(2k+2)} = \frac{1}{2}x^2 - \frac{1}{12}x^4 + \frac{1}{30}x^6 - \ldots$

   (b) $(-1, 1)$