

Math 126 Winter 2016
Final Exam Answers

1. (a) $\theta = \cos^{-1} \left(\frac{7}{\sqrt{13}\sqrt{14}} \right) \approx 59^\circ$
 (b) (5, 4, 6)
2. (a) $t = \frac{1}{4}$ and $t = 1$
 (b) $\left(-\frac{3\pi}{10}, \frac{3}{10}, 0 \right)$
3. (a) $y = -3\sqrt{3} \left(x - \frac{5\sqrt{3}}{4} \right) + \frac{5}{4}$
 (b) $\text{area} = 2 + \frac{3}{8}\pi - \frac{17}{16}\sqrt{3}$
4. (a) $f_x(x, y) = yx^{y-1}$, $f_y(x, y) = x^y \ln(x)$
 (b) $z = -\frac{1}{5}(x - 2) + \frac{1}{5}(y - 2)$
 (c) One possible answer: $f(x, y) = x^2 - y^2$ (There are many others.)
5. (a) $\text{volume} = 36\pi(\sqrt{2} - 1)$
 (b) $\int_0^1 \int_x^1 \sqrt{1+y^2} dy dx = \int_0^1 \int_0^y \sqrt{1+y^2} dx dy = \frac{1}{3}(2\sqrt{2} - 1)$
6. (a) $\mathbf{r}(t) = \langle 2 \cos t + 3 \sin t, \cos t - 6 \sin t, -\sqrt{40} \cos t + \sqrt{40} \rangle$
 (b) $a_N = \sqrt{45}$
7. (a) $\sum_{k=0}^{\infty} \left[\frac{(-1)^k 3^{k+1}}{k+1} + \frac{(-2)^k}{k!} - 4(-5)^k \right] x^{k+1}$
 (b) $\left(-\frac{1}{5}, \frac{1}{5} \right)$
 (c) $T_3(x) = \frac{9}{2}x^3$
8. (a) $T_2(x) = (1 - \pi)(x - 1) + \frac{(\pi^2+2)}{2}(x - 1)^2$
 (b) On the interval $\left[\frac{1}{2}, \frac{3}{2} \right]$,

$$|f'''(x)| = |-\pi^3 \cos(\pi x) + \pi^3 \sin(\pi x) + 6(2-x)^{-4}| \leq 2\pi^3 + 96 \leq 162.$$

One possibility: take $M = 162$.

Then $\text{error} = |T_2(x) - f(x)| \leq \frac{162}{6} \left(\frac{1}{2} \right)^3 = 3.375$ (Other correct answers are possible.)

(c) Using M from part (b),

$$|T_2(x) - f(x)| \leq \frac{162}{6} |x - 1|^3 = 27|x - 1|^3 \leq 0.001 \text{ if } |x - 1| \leq \frac{1}{30} \approx 0.033$$

$I = [0.967, 1.033]$ (Other answers are possible.)