MATH 126 – FINAL EXAM Answers
WINTER 2018

1. (a) (NOTE: Answer is not unique.) $x(t) = 3 + 2t, y(t) = 2, z(t) = 2 + t$
   (b) $\mathbf{proj}_{\overrightarrow{PQ}}\overrightarrow{PR} = \left\langle -\frac{12}{5}, 0, -\frac{6}{5} \right\rangle$
   (c) $\left(-\frac{7}{5}, 2, -\frac{1}{5}\right)$

2. (a) $\mathbf{r}'(t) = \langle 2t, 2, t^2 \rangle$, $\mathbf{r}''(t) = \langle 2, 0, 2t \rangle$
   (b) (NOTE: Answer is not unique.) $x(t) = 2t,\ y(t) = 2 + 2t,\ z(t) = \frac{1}{3} + t$
   (c) $\kappa(1) = \frac{2}{9}$

3. (a) $(0, 1, 1)$ and $(0, 2, 2)$
   (b) $z - 3 = -\frac{25}{4}(x - 0) + \frac{1}{4}(y - 5)$

4. max$=\sqrt{3}$ at the critical point $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

   min $= 1$ at each of the "corners": $(0, 0)$, $(1, 0)$, and $(1, 0)$

5. HINT: Total volume of S is 2. You need the value of m such that $\displaystyle\int_0^{1/m}\int_{mx}^{1} 12xy^2\,dy\,dx = 1.$

   ANSWER: $m = \sqrt{\frac{6}{5}}$

6. HINT: Convert to polar coordinates. Please.
   ANSWER: $\frac{16}{5}\left(2 - \sqrt{2}\right)$

7. (a) $T_6(x) = \dfrac{1}{8} + \dfrac{x^3}{8^2} + x^5 + \dfrac{x^6}{8^3}$
   (b) $-2 < x < 2$

8. (a) $T_1(x) = 2 + \frac{1}{2}(x - 1)$
   (b) $\sqrt{3.25} = g(0.5) \approx T_1(0.5) = 1.75$
   (c) HINT: $|g''(x)| = \dfrac{3}{(3 + x^2)^{3/2}}$. This is positive and decreasing on $[0.5, 1]$.

   The smallest upper bound for $|g''|$ on this interval is $\dfrac{3}{(3.25)^{3/2}}$.

   Larger values of M are also ok.
   For example: $|g''(x)| \le \dfrac{3}{3.25^{3/2}} \le \dfrac{3}{3^{3/2}} = \dfrac{1}{\sqrt{3}} < 1.$

   ANSWER: (Using $M = 1$.) $|f(0.5) - T_1(0.5)| \le \frac{1}{8}$