Math 126 G - Autumn 2017
Midterm Exam Number Two
November 16, 2017

Name: Saul Uxin   Student ID no. : hey, that's private!

Signature:

1	9	9
2	10	10
3	9	9
4	16	16
5	16	16
Total	60	60

Whoa!

- This exam consists of FIVE problems on SIX pages, including this cover sheet.
- Show all work for full credit. Show no work for zero credit.
- You do not need to simplify your answers.
- If you use a trial-and-error or guess-and-check method when a more rigorous method is available, you will not receive full credit.
- Write all of your work on the exam itself. If you use the back of the page, please indicate that you have done so!
- You may use a TI-30X IIS on this exam. No other electronic devices are allowed.
- You may use one hand-written double-sided 8.5″ by 11″ page of notes.
- You have 50 minutes to complete the exam.

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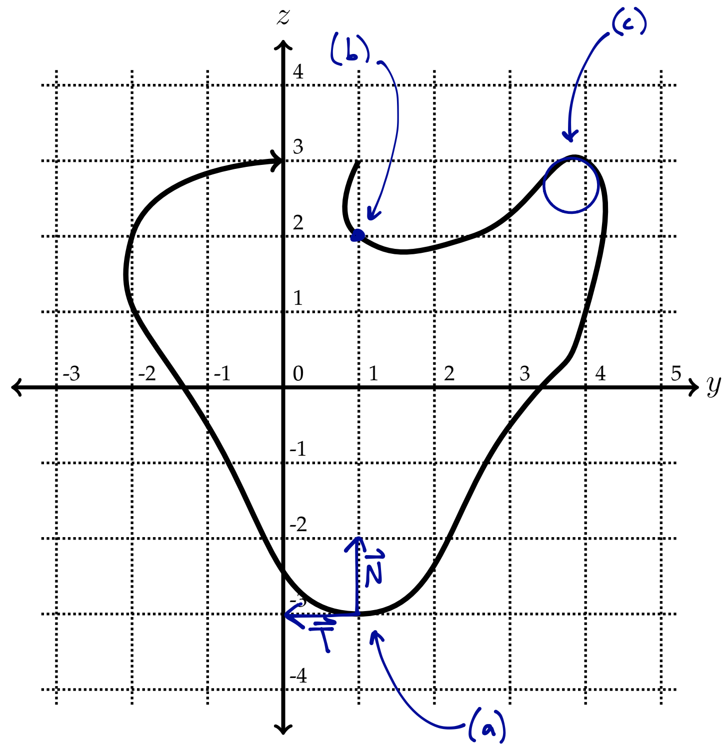
1. **[3 points per part]** Suppose $\mathbf{r}(t) = \langle 0, y(t), z(t) \rangle$ for some functions $y(t)$, $z(t)$.
Here's a picture of the space curve of $\mathbf{r}(t)$ in the yz-plane.



(a) Compute **T**, **N**, and **B** at the point $(0, 1, -3)$.

$\vec{T} = \boxed{\langle 0, -1, 0 \rangle}$ the direction the curve is pointing.

$\vec{N} = \boxed{\langle 0, 0, 1 \rangle}$ the direction it's turning.

$\vec{B} = \vec{T} \times \vec{N} = \boxed{\langle -1, 0, 0 \rangle}$

(b) Find another point on the graph where **B** exists, but is different from the vector you found in part (a).

$\vec{B}$ will be $\langle 1, 0, 0 \rangle$ when the curve turns the other way (counter-clockwise), e.g. @ $\boxed{(0, 1, 2)}$.

(c) Estimate the curvature at the point $(0, 4, 3)$.
(You don't have to be very accurate, but you should show your reasoning.)

$\kappa = \frac{1}{R}$ radius of a circle that fits along the curve.

$R \approx \frac{1}{3}$, so $\boxed{\kappa \approx 3}$

2. **[10 points]** Consider the surface

$$z = f(x, y) = y \arctan(x) + xy^2 e^y.$$

Find the equation of the plane tangent to this surface at the point $(1, 4, f(1, 4))$.

We need $f(1,4)$, $f_x(1,4)$, and $f_y(1,4)$:

$$f(1,4) = 4\arctan(1) + 16e^4 = \pi + 16e^4$$

$$f_x(x,y) = \frac{y}{1+x^2} + y^2 e^y$$

$$f_x(1,4) = 2 + 16e^4$$

$$f_y(x,y) = \arctan(x) + 2xye^y + xy^2e^y$$

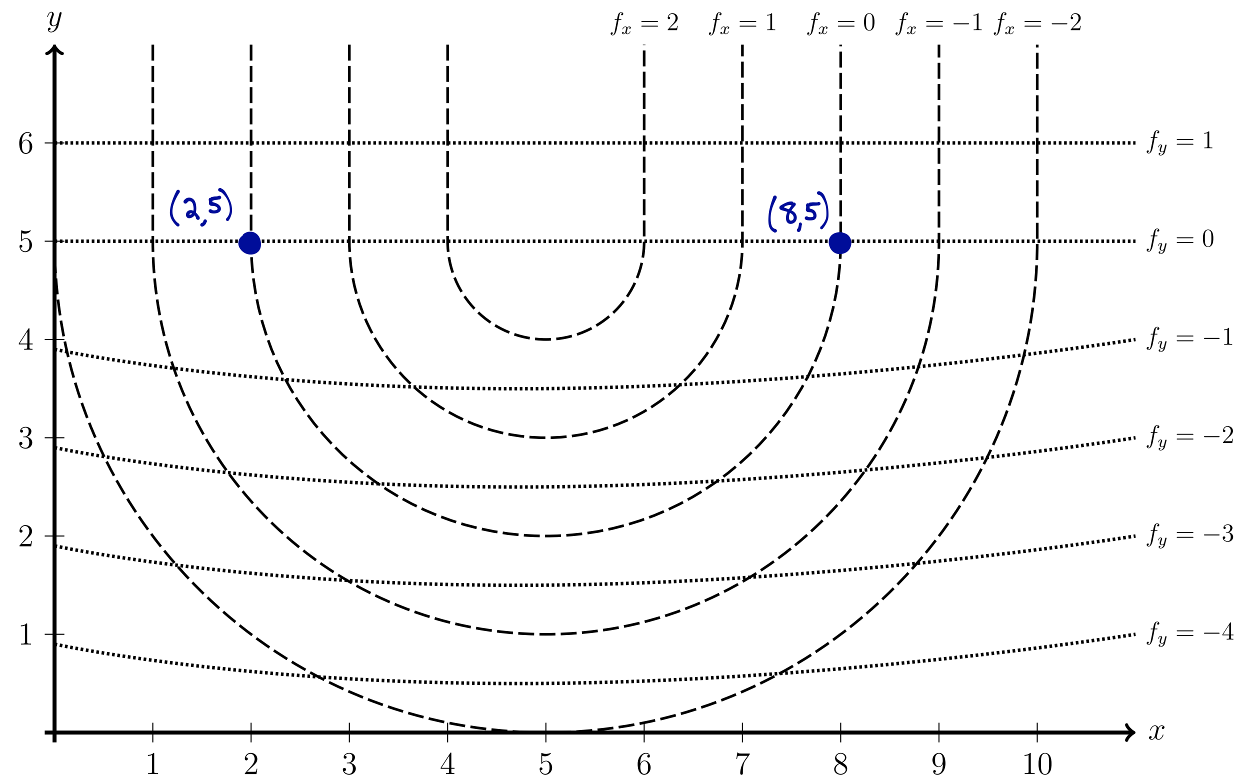
$$f_y(1,4) = \frac{\pi}{4} + 24e^4$$

Tangent plane:

$$z = f(1,4) + f_x(1,4)(x-1) + f_y(1,4)(y-4)$$

$$\boxed{z = \pi + 16e^y + (2+16e^4)(x-1) + \left(\frac{\pi}{4} + 24e^4\right)(y-4)}$$

3. **[9 points]** Below are the **level curves** of the **partial derivatives** of a function $f(x, y)$.



(a) Find the critical points of f.

Where $f_x = 0$ and $f_y = 0$: at $\boxed{(2,5)}$ and $\boxed{(8,5)}$

(b) Identify each critical point as a local maximum, local minimum, or saddlepoint.

At $(2,5)$: f_x increases in the x-direction $\rightarrow f_{xx} > 0$

f_x is constant in the y-direction $\rightarrow f_{xy} = 0$

f_y increases in the y-direction $\rightarrow f_{yy} > 0$

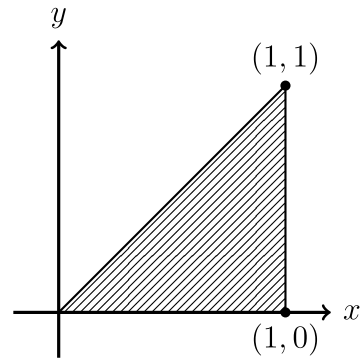
$D = f_{xx} f_{yy} - [f_{xy}]^2 > 0$

$\boxed{\text{local min at } (2,5)}$

Similarly, at $(8,5)$: $f_{xx} < 0$, $f_{xy} = 0$, $f_{yy} > 0$ $\rightarrow D < 0 \rightarrow$ $\boxed{(8,5) \text{ is a saddlepoint}}$

4. **[16 points]** Consider the function $f(x, y) = x + xy - 3y^2$.

Find the maximum and minimum values of $f(x, y)$ on the triangle D pictured below:



Critical points: $f_x(x,y) = 1 + y = 0$, $f_y(x,y) = x - 6y = 0$ $\rightarrow$ $\underbrace{y = -1,\ x = -6}_{\text{not in domain.}}$

Boundary:

Vertices $\boxed{(0,0)}$, $\boxed{(1,0)}$, $\boxed{(1,1)}$

Bottom: $y = 0$, $f(x,0) = x$, no interior extrema.

Right: $x = 1$, $f(1,y) = 1 + y - 3y^2$ $\xrightarrow{\frac{d}{dy}}$ $1 - 6y = 0 \rightarrow y = \frac{1}{6} \rightarrow \boxed{\left(1, \frac{1}{6}\right)}$

Upper-left: $y = x$, $f(x,x) = x + x^2 - 3x^2$ $\xrightarrow{\frac{d}{dx}}$ $1 - 4x \rightarrow x = \frac{1}{4} \rightarrow \boxed{\left(\frac{1}{4}, \frac{1}{4}\right)}$

Check:

$f(0,0) = 0$

$f(1,0) = 1$

$f(1,1) = -1$ ← min

$f\left(1, \frac{1}{6}\right) = \frac{13}{12}$ ← max

$f\left(\frac{1}{4}, \frac{1}{4}\right) = \frac{1}{8}$

5. **[8 points per part]** For each f and D shown below, compute $\iint\limits_D f(x,y)\,dA$.

(a) $f(x,y) = x^5 \sin(x^3 y)$

D is the rectangle $[1,2] \times [0,3]$.

$$\int_1^2 \int_0^3 x^5 \sin(x^3 y)\,dy\,dx = \int_1^2 \int_0^{3x^3} x^2 \sin(u)\,du\,dx = \int_1^2 \left(-x^2\cos(u)\right]_{u=0}^{u=3x^3} dx$$

$u = x^3 y$

$du = x^3\,dy$

$$= \int_1^2 \left(-x^2\cos(3x^3) + x^2\right)dx = \int_1^2 -x^2\cos(3x^3)\,dx + \left(\frac{1}{3}x^3\right]_1^2$$

$u = 3x^3$

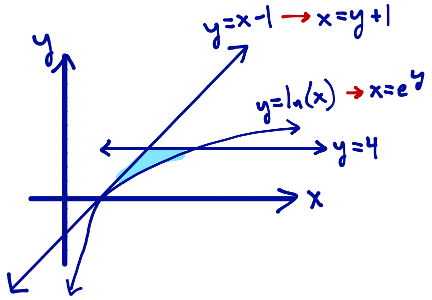
$du = 9x^2$

$$= \frac{-1}{9}\int_3^{24} \cos(u)\,du + \left(\frac{8}{3} - \frac{1}{3}\right)$$

$$= \frac{-1}{9}\left(\sin(u)\right]_3^{24} + \frac{7}{3} = \boxed{\frac{-1}{9}\left(\sin(24) - \sin(3)\right) + \frac{7}{3}}$$

(b) $f(x,y) = y$

D is the region bounded by $y = 4$, $y = \ln(x)$, and $y = x - 1$.



$$\int_0^4 \int_{y+1}^{e^y} y\,dx\,dy = \int_0^4 \left(xy\right]_{x=y+1}^{x=e^y} dy$$

$$= \int_0^4 \left(ye^y - y^2 - y\right)dy = \int_0^4 ye^y\,dy - \left(\frac{1}{3}y^3 + \frac{1}{2}y^2\right]_0^4$$

$u = y \quad dv = e^y\,dy$

$du = dy \quad v = e^y$

$$= ye^y\Big]_0^4 - \int_0^4 e^y\,dy - \left(\frac{64}{3} + 8\right) = 4e^4 - \left(e^4 - 1\right) - \frac{88}{3} = \boxed{3e^4 - \frac{85}{3}}$$