1. Consider the function $f(x, y) = 2xy^2 - y^3 + x\sqrt{y} - 4$.

   (a) **[7 points]** Write the linearization $L(x, y)$ for f at the point $(3, 4)$.

$f_x(x,y) = 2y^2 + \sqrt{y}$

$f_y(x,y) = 4xy - 3y^2 + \frac{x}{2\sqrt{y}}$

$f(3,4) = 34$

$f_x(3,4) = 34$

$f_y(3,4) = \frac{3}{4}$

$L(x,y) = f(3,4) + f_x(3,4)(x-3) + f_y(3,4)(y-4)$

$\boxed{L(x,y) = 34 + 34(x-3) + \frac{3}{4}(y-4)}$

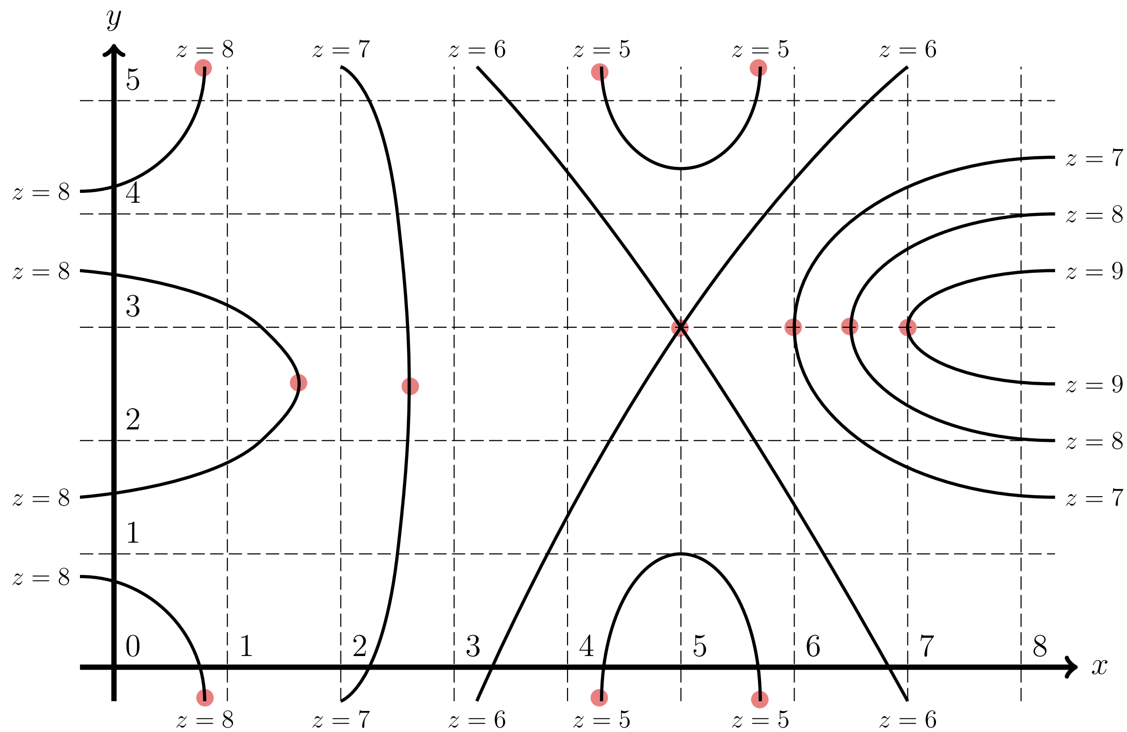
   (b) **[3 points]** Approximate a value of y such that $f(3.01, y) = 33.95$.

$33.95 = 34 + 34(3.01-3) + \frac{3}{4}(y-4)$

$-0.39 = \frac{3}{4}(y-4)$

$\boxed{y = 3.48}$

2. **[3 points per part]** Here are the level curves of the surface $z = f(x, y)$.



(a) Name three points at which $f_y(x, y) = 0$.

Any of the red points in the graph above

(b) Write the equation for the tangent plane to the surface $z = f(x, y)$ at the point $(5, 3, 6)$.

This is a saddle point! $f_x(5,3) = f_y(5,3) = 0$,

so the tangent plane is just $\boxed{z=6.}$

(c) Estimate the value of $\int_3^5 \int_2^4 f(x, y)\, dy\, dx$. (Circle one answer.)

Less than 0	Between 0 and 10	Between 10 and 20
Between 20 and 30	Between 30 and 40	Greater than 40

volume of a solid

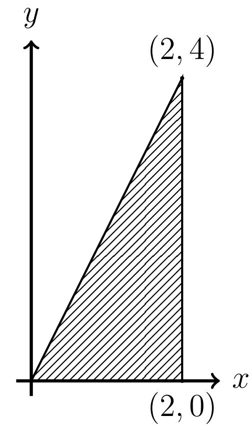
base is a square of area 4.

height is between $z=5$ and $z=7$.

So volume is between $4\cdot 5$ and $4\cdot 7$.

3. **[12 points]** Consider the function $f(x, y) = x^3 + xy - y^2$.

Find the absolute maximum and minimum values of $f(x, y)$ on the triangle below:



Critical points:

$f_x(x,y) = 3x^2 + y = 0$ → $3(2y)^2 + y = 0$

$f_y(x,y) = x - 2y = 0$ → $x = 2y$

$12y^2 + y = 0$

$y(12y+1) = 0$

$y = 0$ or $\boxed{\cancel{y = \frac{-1}{12}}}$

$x = 0$ not in domain

One critical point: $(0,0)$.

Bottom edge: $y = 0$

$f(x,0) = x^3$, increasing, no local extrema.

Right edge: $x = 2$

$f(2,y) = 8 + 2y - y^2$

$\frac{d}{dy} \downarrow$

$2 - 2y = 0$

$y = 1$ → check $(2,1)$

Top-left edge: $y = 2x$

$f(x,2x) = x^3 + 2x^2 - 4x^2$

$\frac{d}{dx} \downarrow$

$3x^2 - 4x = 0$

$x(3x-4) = 0$

$x = 0$, $x = \frac{4}{3}$

So also check $\left(\frac{4}{3}, \frac{8}{3}\right)$.

Also, check all corners.

So: $f(0,0) = 0$

$f(2,0) = 8$

$f(2,1) = \boxed{9}$ ← max

$f(2,4) = 0$

$f\left(\frac{4}{3}, \frac{8}{3}\right) = \boxed{\frac{-32}{27}}$ ← min

4. **[7 points per part]** Evaluate each integral.

(a) $\displaystyle\int_0^1 \int_0^3 y\sqrt{1+xy}\,dy\,dx$

$$= \int_0^3 \int_0^1 y\sqrt{1+xy}\,dx\,dy = \int_0^3 \int_1^{1+y} \sqrt{u}\,du\,dy = \int_0^3 \frac{2}{3}\left(u^{3/2}\right)\Big]_1^{1+y} dy = \frac{2}{3}\int_0^3 \left((1+y)^{3/2} - 1\right) dy$$

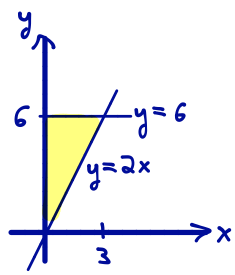
$u = 1+xy$

$du = y\,dx$

$$= \frac{2}{3}\left(\frac{2(1+y)^{5/2}}{5} - y\right)\Bigg]_0^3 = \frac{2}{3}\left(\frac{2}{5}(32) - 3\right) - \frac{2}{3}\left(\frac{2}{5}\right)$$

$$= \boxed{\frac{94}{15}}$$

(b) $\displaystyle\int_0^3 \int_{2x}^6 e^{y^2}\,dy\,dx$



$$= \int_0^6 \int_0^{\frac{y}{2}} e^{(y^2)}\,dx\,dy$$

$$= \int_0^6 \left(x e^{y^2}\right)\Big]_0^{\frac{y}{2}} dy = \int_0^6 \frac{y}{2} e^{y^2} dy = \int_0^{36} \frac{1}{4} e^{u} du = \frac{1}{4}\left(e^{u}\right)\Big]_0^{36}$$

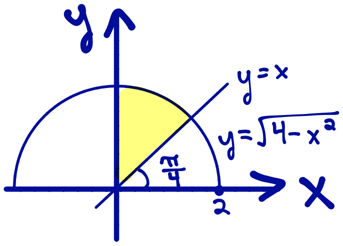
$u = y^2$

$du = 2y\,dy$

$$= \boxed{\frac{1}{4}\left(e^{36} - 1\right)}$$

5. **[7 points]** Oh wow, another integral! Nice!

Evaluate $\displaystyle\int_0^{\sqrt{2}}\int_x^{\sqrt{4-x^2}} \sin(x^2+y^2)\,dy\,dx.$



$$\int_{\pi/4}^{\pi/2}\int_0^2 \sin(r^2)\,r\,dr\,d\theta = \int_{\pi/4}^{\pi/2}\int_0^4 \frac{1}{2}\sin(u)\,du\,d\theta$$

$u=r^2$
$du=2r\,dr$

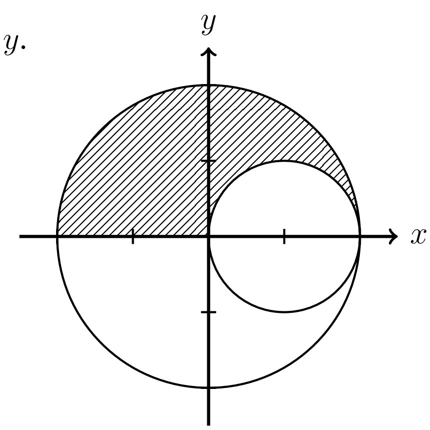
$$= \int_{\pi/4}^{\pi/2}\left(\frac{-1}{2}\cos(u)\right)\Big]_{u=0}^{u=4}d\theta = \int_{\pi/4}^{\pi/2}\left(\frac{-1}{2}\cos(4)+\frac{1}{2}\right)d\theta = \left(\frac{-1}{2}\cos(4)+\frac{1}{2}\right)\theta\Big]_{\pi/4}^{\pi/2} =$$

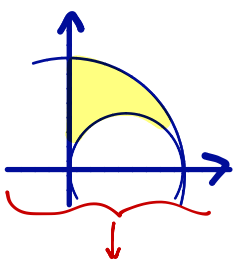
$$= \left(\frac{-1}{2}\cos(4)+\frac{1}{2}\right)\left(\frac{\pi}{2}-\frac{\pi}{4}\right) = \boxed{\frac{\pi}{8}\left(1-\cos(4)\right)}$$

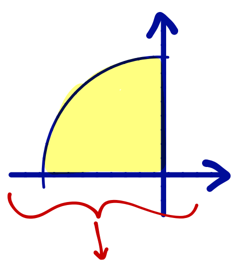
6. **[8 points]** Let D be the region pictured below, bounded by the two circles $x^2+y^2=4$, $(x-1)^2+y^2=1$, and the x-axis.

A lamina in the shape of D has density function $\rho(x,y)=y$.

Compute the mass of the lamina.







$$= \int_0^{\pi/2}\int_{2\cos\theta}^{2} r^2\sin\theta\,dr\,d\theta + \int_{\pi/2}^{\pi}\int_0^2 r^2\sin\theta\,dr\,d\theta$$

$$= \int_0^{\pi/2}\frac{1}{3}\left(r^3\sin\theta\right)\Big]_{2\cos\theta}^{2}d\theta + \int_{\pi/2}^{\pi}\frac{1}{3}\left(r^3\sin\theta\right)\Big]_0^2 d\theta = \frac{1}{3}\left(\int_0^{\pi/2}\left(8-8\cos^3\theta\right)\sin\theta\,d\theta + \int_{\pi/2}^{\pi}8\sin\theta\,d\theta\right)$$

$u=\cos\theta$
$du=-\sin\theta\,d\theta$

$$= \frac{1}{3}\left(8\int_1^0\left(u^3-1\right)du + \left(-8\cos\theta\right)\Big]_{\pi/2}^{\pi}\right) = \frac{1}{3}\left(8\left(\frac{u^4}{4}-u\right)\Big]_1^0 - 8(-1)\right) = \frac{8}{3}\left(\frac{3}{4}+1\right) = \boxed{\frac{14}{3}}$$