Math 126, Section C - Winter 2015
Midterm II
March 3, 2015

Name: _____

Student ID Number: _____

Section: CA 11:30-12:20 by Sam ☐ CB 12:30-1:20 by Sam ☐

CC 11:30-12:20 by Ru-Yu ☐ CD 12:30-1:20 by Ru-Yu ☐

exercise	**possible**	**score**
1	12	
2	14	
3	12	
4	12	
total	50	

- Check that this booklet has all the exercises indicated above.
- TURN OFF YOUR CELL PHONE.
- Write your name and your student ID.
- This is a 50 minute test.
- You may use a scientific calculator and one 8.5×11 inch sheet of (two-sided) handwritten notes. All other electronic devices (including graphing calculators) are forbidden.
- Unless otherwise indicated, your answers should be exact instead of decimal approximations. For example $\frac{\pi}{4}$ is an exact answer and is preferable to its decimal approximation 0.78.
- Unless otherwise indicated, show your work and justify all your answers.
- Do not use scratch paper! Do your scratch work at the bottom of the page or on the back side of the preceding sheet.

Exercise 1 (12 points).

The equation

$$xyz + 2x^2y^2 - 3y^2z^3 = -2$$

determines a surface in $\mathbb{R}^3$. Find the tangent plane to the surface at the point $(1, 2, 1)$.

Hint: Use implicit differentiation.

Exercise 2 (10+2+2=14 points).

Consider the function

$$f(x,y) = \frac{1}{x^2+1} - xy + y$$

a) Determine and classify all critical points of f.

b) We want to understand better, how the function f behaves for points (x,y) that have a large magnitude.

i) Find a vector function $\vec{r}(t) = (x(t),y(t))$ so that $f(x(t),y(t)) \to \infty$ as $t \to \infty$.

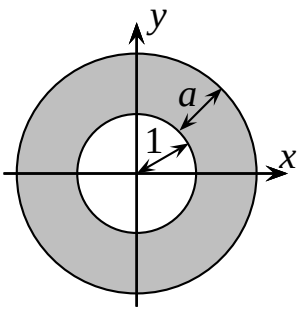
ii) Find a vector function $\vec{r}(t) = (x(t),y(t))$ so that $f(x(t),y(t)) \to -\infty$ as $t \to \infty$.

Exercise 3 (12 points).

We want to construct a lamina in form of a ring that has inner radius 1 and thickness a. If the density of the lamina is given by

$$\rho(x,y) = x^4 + 2x^2y^2 + y^4,$$

then how do we need to choose a so that the mass of the lamina will be exactly $728 \cdot \frac{\pi}{3}$?



Exercise 4 (12 points).

Compute the integral

$$\int_{-4}^{0}\int_{\sqrt{-y}}^{2} y\cdot\sqrt{5+x^5}\,dx\,dy$$

Hint: Reverse the order of integration.