# Examples on Typesetting Commutative Diagrams Using XY-pic

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*Edition 1*

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This printout provides examples on typesetting commutative diagrams using XY-pic's `\xymatrix{...}` command which view commutative diagrams as "matrix-like diagrams". The printout is an attempt to introduce the complete newcomer to XY-pic.

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# Contents

# Chapter 1

# On Typesetting Commutative Diagrams

## 1.1 Introduction

In category theory, “commutative diagrams” are the categorists ways to illustrate equations and universal properties. Here is an example:

*Let* $\mathfrak{E}$ *be a topos, and let* $\Omega$ *be its subobject-classifier. A* Lawvere-Tierney topology *on* $\mathfrak{E}$ *is a map* $j : \Omega \longrightarrow \Omega$ *in* $\mathfrak{E}$ *such that the following three diagrams commute.*

$$\begin{tikzcd} 1 \arrow[r, "\mathrm{true}"] \arrow[dr, "\mathrm{true}"'] & \Omega \arrow[d, "j"] \\ & \Omega \end{tikzcd} \qquad \begin{tikzcd} \Omega \arrow[r, "j"] \arrow[dr, "j"'] & \Omega \arrow[d, "j"] \\ & \Omega \end{tikzcd} \qquad \begin{tikzcd} \Omega \times \Omega \arrow[r, "\wedge"] \arrow[d, "j \times j"'] & \Omega \arrow[d, "j"] \\ \Omega \times \Omega \arrow[r, "\wedge"'] & \Omega \end{tikzcd}$$

$$\Big(j \circ \mathrm{true} = \mathrm{true}\Big) \qquad \Big(j \circ j = j\Big) \qquad \Big(j \circ \wedge = \wedge \circ (j \times j)\Big)$$

There are a few programs to draw commutative diagrams. Two of the best are XY-pic and PSTricks.

XY-pic package is © by its authors as free software for typesetting graphs and diagrams such as arrows, curves, frames, directed graphs, paths, polygons, knots, and commutative diagrams and 2-cell structures as in 2-categories. It was written by:
Kristoffer Rose: `<Kristoffer.Rose@ENS-Lyon.FR>`, and
Ross Moore: `<ross@ics.mq.edu.au>`.

For complete information on XY-pic diagrams in general, please refer to the XY-guide and XY-reference manual, which can be downloaded and printed out, from one of the following sites:
`<http://www.ens-lyon.fr/~krisrose/Xy-pic.html>`
`<http://www.ics.mq.edu.au/~ross/Xy-pic.html>`

### Overall Structure of a Document

XY-pic works with most formats (including LaTeX, AMS-LaTeX, AMS-TeX, and plain TeX). It must be loaded into your format working memory.
Here is **a sample** of the overall structure of a LaTeX Document:

```
\documentclass[12pt]{report}
\usepackage{geometry,amsthm,graphics,amssymb,amsmath,enumerate,latexsym,
tabularx,shapepar}
\usepackage[all,2cell,dvips]{xy} \UseAllTwocells \SilentMatrices
\begin{document}
            .
            .
            .
            .
            .
            .
\end{document}
```

The control sequences
`\usepackage[all,2cell,dvips]{xy} \UseAllTwocells \SilentMatrices`
are the ones I used to produce this document.

## 1.2 The Basic Construction

`\xymatrix{...}` command views a commutative diagram as a "matrix-like diagram"; a matrix that has "vertices" or "entries". To get an idea, suppose we want to typeset the diagram

$$\begin{tikzcd} A \arrow[r] \arrow[d] & B \arrow[d] \arrow[ld] \\ C \arrow[r] & D \end{tikzcd}$$

First, view the vertices of the diagram

$$\begin{matrix} A & B \\ & \\ C & D \end{matrix}$$

as entries of a matrix. Any two entries can serve as a source and target for an arrow. An arrow is set by typing the command that produces that arrow *right after* the source entry where the arrow starts. Such command takes the form `\ar[direction]` where the variable "direction" takes one value as illustrated by the diagram:

$$\begin{tikzcd} lu & u & ru \\ l & \bullet \arrow[lu] \arrow[u] \arrow[ru] \arrow[l] \arrow[r] \arrow[ld] \arrow[d] \arrow[rd] & r \\ ld & d & rd \end{tikzcd}$$

For example, `\ar[r]` produces a right (east) arrow from its source, and `\ar[rd]` produces a right-down (southeast) arrow from its source. In addition, other combinations of directions are possible. For example, `\ar[rru]` will produce a right-right-up (east-northeast) arrow from its source, and `\ar[rdd]` will produce a right-down-down (south-southeast) arrow from its source, and so on.

<u>Note:</u> *It has the same affect to use [ru] or [ur]. Similarly, all other combination of directions.*

*Now let us apply what we have said.* Our commutative diagram

$$\begin{tikzcd} A \arrow[r] \arrow[d] & B \arrow[d] \arrow[ld] \\ C \arrow[r] & D \end{tikzcd}$$

can be typed as

```
\xymatrix{ A \ar[r] \ar[d] & B \ar[d] \ar[ld]\\
           C \ar[r] & D}
```

Here is a second example: the diagram

$$\begin{tikzcd} A & B \arrow[ldd] \\ C & D \\ E & F \end{tikzcd}$$

can be produced by

```
\xymatrix{ A & B \ar[ldd]\\
           C & D\\
           E & F}
```

It's all as simple as that, more or less. Notice how the special `&` character separates the entries in any one row, and the \\ separates different rows. To center the diagram by itself in a page you may want to enclose it by `\[ ... \]` or `$$ ... $$`.

If you use `\ar[direction]` where is there is no target, that is, if you point an arrow outside the `\xymayrix{ ... }` grid then LaTeX will respond by giving an error message to the terminal screen.

In some diagrams, there is no vertix in an entry. Empty vertices may be omitted, but we still need to use the `&` to separate the columns in order for the arrows be pointing somewhere. In general, empty vertices at the end of rows may be omitted. Here is an example:

$$\begin{tikzcd} 1 \arrow[r] \arrow[rd] & \Omega \arrow[d] \\ & {} \end{tikzcd}$$

```
\xymatrix{ 1  \ar[r] \ar[rd] &\Omega \ar[d]\\
                             &  }
```

You may prefer to use {} or \ to replace an empty vertix.

## 1.3 Arrows' Labels

Arrows can be labeled. The position of labels is specified by `^`, `_`, for the position, and `|` for breaking an arrow with a label. The dash `-` used, if needed, to center labels. Here are examples. (Notice the reversed meaning of `^` and `_` when arrows are reversed from right to left and from down to up.)

- for right arrows, `\ar[r]|f` produces $\xrightarrow{\quad f \quad}$
- for right arrows, `\ar[r]^f_g` produces $\xrightarrow[g]{f}$
- for left arrows, `\ar[l]^f_g` produces $\xleftarrow[f]{g}$
- for right down arrows, `\ar[rd]^f` produces
$$\begin{tikzcd} \cdot \arrow[dr, "f"] & \\ & \cdot \end{tikzcd}$$
- for left up arrows, `ar[lu]_f` produces
$$\begin{tikzcd} \cdot & \\ & \cdot \arrow[ul, "f"'] \end{tikzcd}$$

Similarly, other-direction arrows.

#### Vertices' Long Names

XY-pic essentially lays down the vertices first, and then superimposes the arrows upon them. The size of the individual vertices and the sizes of the column and row gaps are taken into account when the vertices are first printed, but their position is not influenced by the arrows, and, in particular, not by any labels on the arrows. This means that a long vertix can causes the arrow label to be positioned not in the middle of the arrow, here is an example:

```
\xymatrix{ M \ar[r]^\theta  & S\otimes  M\otimes S }
```

$$M \xrightarrow{\qquad\theta} S \otimes M \otimes S$$

This is because XY-pic places the label between the centers of the source and target objects. To correct this, so that the label can centered in the arrow, add a dash `-` right before the label, as follows:

```
\xymatrix{ M \ar[r]^-\theta  & S\otimes  M\otimes S }
```

$$M \xrightarrow{\theta} S \otimes M \otimes S$$

### Arrows' Long Labels

Also, a long arrow's label causes the diagram to be cramped. For example,

```
\[
\xymatrix{(d_0)_!(d_0)_!d^*_2d^*_1
\ar[r]^{\tilde{d_1}=(d_0)_!\epsilon^{d_1}d_1^*}
& (d_0)_!d_1^*}
\]
```

$$(d_0)_!(d_0)_!d^*_2d^*_1 \xrightarrow{\tilde{d_1}=(d_0)_!\epsilon^{d_1}d_1^*} (d_0)_!d_1^*$$

This problem can be solved by adding extra columns of 'empty' vertices in the diagram. For example, adding two columns, and using the dash `-` produce

```
\[
\xymatrix{(d_0)_!(d_0)_!d^*_2d^*_1
\ar[rrr]^-{\tilde{d_1}=(d_0)_!\epsilon^{d_1}d_1^*}
&&& (d_0)_!d_1^*}
\]
```

$$(d_0)_!(d_0)_!d^*_2d^*_1 \xrightarrow{\qquad\tilde{d_1}=(d_0)_!\epsilon^{d_1}d_1^*\qquad} (d_0)_!d_1^*$$

Another solution is by increasing the column gap between the two vertices. For example, adding the option `@C=2.5pc`[1], just after the command `\xymatrix` :

```
\[
\xymatrix @C=2.5pc {(d_0)_!(d_0)_!d^*_2d^*_1
\ar[rrr]^-{\tilde{d_1}=(d_0)_!\epsilon^{d_1}d_1^*}
&&& (d_0)_!d_1^*}
\]
```

$$(d_0)_!(d_0)_!d^*_2d^*_1 \xrightarrow{\qquad\quad\tilde{d_1}=(d_0)_!\epsilon^{d_1}d_1^*\qquad\quad} (d_0)_!d_1^*$$

[1] See the section about measurements in TEX.

### Moving Labels

Labels can be slided along an arrow, if needed, by adding the option `(number)` just before the label, Here are examples:

| | |
|---|---|
| `\[\xymatrix {M\ar [r]^\theta &S}\]` | $M \xrightarrow{\theta} S$ |
| `\[\xymatrix {M\ar [r]^(.75)\theta &S }\]` | $M \xrightarrow{\quad\ \theta} S$ |
| `\[\xymatrix {M\ar [r]^(.5)\theta &S}\]` | $M \xrightarrow{\theta} S$ |
| `\[\xymatrix {M\ar [r]^(.25)\theta &S}\]` | $M \xrightarrow{\theta\quad\ } S$ |
| `\[\xymatrix {M\ar [r]^(-.5)\theta &S}\]` | $^{\theta}\quad M \longrightarrow S$ |
| `\[\xymatrix {M\ar [r]^(1.5)\theta &S}\]` | $M \longrightarrow S \quad ^{\theta}$ |

In a particular situation, trial and error may be helpful in choosing a suitable position for a label.

## Left and Right Shifts of vertices

The position of a particular entry in a commutative diagram can be modified, if needed, by entering `**[l]` or `**[r]` just before the entry. The first produces a left shift in the object by almost one-half of its width, and the second produces a right shift in the object by almost one-half of its width. Compare the following two diagrams:

```
\[
\xymatrix{
A\oplus B \oplus C\ar[r]^f \ar[d] &B \ar[d]\\
A\ar[r]_g & A\oplus B\oplus C}
\]
```

$$\begin{tikzcd} A \oplus B \oplus C \arrow[r, "f"] \arrow[d] & B \arrow[d] \\ A \arrow[r, "g"'] & A \oplus B \oplus C \end{tikzcd}$$

```
\[
\xymatrix{
**[l]A\oplus B \oplus C\ar[r]^f \ar[d] &B \ar[d]\\
A\ar[r]_g & **[r]A\oplus B\oplus C}
\]
```

$$\begin{tikzcd} A \oplus B \oplus C \arrow[r, "f"] \arrow[d] & B \arrow[d] \\ A \arrow[r, "g"'] & A \oplus B \oplus C \end{tikzcd}$$

Notice the affect on the horizontal arrows.

## 1.4 Length Measurements in TEX

To change size of diagrams or text in TEX, and its packages, we may utilize the units by which TEX can measure lengths. Here is a table to give an idea of the comparative sizes:

| Unit | Illustration | Comments |
|---|---|---|
| 1 inch (in) | | |
| 1 centimeter (cm) | | |
| 1 millimeter (mm) | | |
| 1 point (pt) | . | `1 point` is about the size of a period. `1 inch=72.27 points`. |
| 1 pica (pc) | | `1 pica`=12 points |
| 1 ex | | `1 ex` is about the height of a small `x` |
| 1 em | | `1 em` is a little smaller than the width of a capital `M`. |

## 1.5 Spacing Between Rows and Columns of a Commutative Diagram

A code such as

```
\xymatrix{
A  \ar[r]^\theta_\approxeq \ar[d]_m &
B  \ar[d]^n\\
C  \ar[r]  & D}
```

will produce a commutative diagram whose size is determined by default.

$$\begin{tikzcd} A \arrow[r, "\theta", "\approxeq"'] \arrow[d, "m"'] & B \arrow[d, "n"] \\ C \arrow[r] & D \end{tikzcd}$$

Its size can be increased by adding the option `@!=2.5pc`, for example, just after the command `\xymatrix`, as follows:

```
\xymatrix@!=2.5pc{
A  \ar[r]^\theta_\approxeq \ar[d]_m &
B  \ar[d]^n\\
C  \ar[r]  & D}
```

$$\begin{tikzcd} A \arrow[r, "\theta", "\approxeq"'] \arrow[d, "m"'] & B \arrow[d, "n"] \\ C \arrow[r] & D \end{tikzcd}$$

As you see, `@!=2.5pc` has a *uniform* affect on length and width. We may choose to adjust spacing between rows and between columns for certain special effects. This can be done by adding the options `@R=some_length` and `@C=some_length` just after the command `\xymatrix`. Here is an example:

```
\xymatrix  @R=.4in  @C=1.5in {
A  \ar[r]^\theta_\approxeq \ar[d]_m &
B  \ar[d]^n\\
C  \ar[r]   & D}
```

$$\begin{tikzcd}[row sep=0.4in, column sep=1.5in] A \arrow[r, "\theta", "\approxeq"'] \arrow[d, "m"'] & B \arrow[d, "n"] \\ C \arrow[r] & D \end{tikzcd}$$

Here is another example: a diagram like

```
\[
\xymatrix{
H_*(B\otimes B)\ar[rr]^-r \ar[rd]_{H_*[r_*]} &&
H_*(B\otimes B)\otimes H_*(B\otimes B) \ar[ld]^{\xi_*}\\
&H_*(B\otimes B\otimes B)}
\]
```

$$\begin{tikzcd} H_*(B\otimes B) \arrow[rr, "r"] \arrow[rd, "H_*[r_*]"'] & & H_*(B\otimes B)\otimes H_*(B\otimes B) \arrow[ld, "\xi_*"] \\ & H_*(B\otimes B\otimes B) & \end{tikzcd}$$

looks better if the spacing between rows is increased, and between columns is decreased, as follows:

```
\[
\xymatrix @R=5pc @C=-0.5pc {
H_*(B\otimes B)\ar[rr]^-r \ar[rd]_{H_*[r_*]} &&
H_*(B\otimes B)\otimes H_*(B\otimes B) \ar[ld]^{\xi_*}\\
&H_*(B\otimes B\otimes B)}
\]
```

$$\begin{tikzcd}[column sep=small, row sep=huge]
H_*(B\otimes B) \arrow[rr, "r"] \arrow[rd, "H_*[r_*]"'] & & H_*(B\otimes B)\otimes H_*(B\otimes B) \arrow[ld, "\xi_*"] \\
& H_*(B\otimes B\otimes B) &
\end{tikzcd}$$

## 1.6 Parallel Arrows

To get a pair of parallel arrows between the same two vertices, move the arrow parallel to itself a small number of distance units. Here is an example:

```
\xymatrix{ X_1 \ar@<+.7ex>[r]\ar@<-.7ex>[r] & X_0}
```

$$X_1 \rightrightarrows X_0$$

where `<+.7ex>` caused the arrow to be moved parallel to itself up by .7 ex, and `<-.7ex>` caused the arrow to be moved parallel to itself down by .7 ex. Similarly, other-direction arrows.

Here is another example in which we need to modify distances in the diagram to improve it. In the following diagram, the arrow and the target object are 'lower' than the source object:

```
\[\xymatrix{\underset{x\in U_{iy}}{\times}A_x\ar[r] & A_y}\]
```

$$\underset{x\in U_{iy}}{\times}A_x \longrightarrow A_y$$

The diagram can be improved by moving the arrow parallel to itself by 0.8 ex as follows:

```
\[\entrymodifiers={!! <0pt, .8ex>+}
\xymatrix{\underset{x\in U_{iy}}{\times}A_x\ar[r] & A_y}\]
```

$$\underset{x\in U_{iy}}{\times}A_x \longrightarrow A_y$$

## 1.7 Arrow Styles

Any arrow has a tail, a shaft, and a head. In the default case

$$A \xrightarrow{f} B$$

the tail is empty, the shaft is a solid line, and the head is an arrow head as shown. XY-pic comes with a large number of arrow styles, where each of the three parts (tail, shaft, head) of an arrow can be changed. The following tables contain a list that I kept for myself as a reference about arrow styles.

| **Objective** and **Input**<br>The commands to produce the diagrams displayed on the right are as follows, respectively: | **Output**<br>(*what you get*) |
|---|---|
| The default case arrow:<br>`\xymatrix { A \ar [r]^f &B}` | $A \xrightarrow{f} B$ |
| Inclusion arrow:<br>`\xymatrix {A \ar @{^{(}->}[r]^f &B}` | $A \xhookrightarrow{f} B$ |
| Epimorphism arrow:<br>`\xymatrix {A \ar @{->>}[r]^f &B}` | $A \overset{f}{\twoheadrightarrow} B$ |
| Monomorphism arrow:<br>`\xymatrix {A\; \ar @{>->}[r]^f &B}` | $A \overset{f}{\rightarrowtail} B$ |
| A function's action on an element:<br>`\xymatrix {x\; \ar @{\|->}[r]^f & f(x)}` | $x \xmapsto{f} f(x)$ |
| Dotted arrow:<br>`\xymatrix { A \ar @{.>}[r]^f &B}` | $A \overset{f}{\dashrightarrow} B$ |
| Dashed arrow:<br>`\xymatrix { A \ar @{-->}[r]^f &B}` | $A \overset{f}{\dashrightarrow} B$ |
| Squiggle arrow:<br>`\xymatrix {A \ar @{~>}[r]^f &B}` | $A \overset{f}{\rightsquigarrow} B$ |

| | |
|---|---|
| Long equal sign:<br>`\xymatrix {A \ar @{=}[r] &B}` | $A = B$ |
| Double dotted arrow:<br>`\xymatrix { A \ar @{:>}[r]^f &B}` | $A \overset{f}{\Rrightarrow} B$ |
| Double arrow:<br>`\xymatrix { A \ar @{=>}[r]^f &B}` | $A \xRightarrow{f} B$ |
| Double arrow:<br>`\xymatrix { A \ar @2{->}[r]^f &B}` | $A \xRightarrow{f} B$ |
| `\xymatrix { A \ar @2{<->}[r]^f &B}` | $A \xLeftrightarrow{f} B$ |
| Triple arrow:<br>`\xymatrix { A \ar @3{->}[r]^f &B}` | $A \overset{f}{\Rrightarrow} B$ |

| | |
|---|---|
| `\xymatrix {A \ar @/^/[r]^f &B}` | $\begin{tikzcd} A \arrow[r, bend left, "f"] & B \end{tikzcd}$ |
| `\xymatrix {A \ar @/^5pc/[r]^f &B}` | $\begin{tikzcd} A \arrow[r, bend left=80, "f"] & B \end{tikzcd}$ |
| `\xymatrix {A \ar @/_/[r]^f &B}` | $\begin{tikzcd} A \arrow[r, bend right, "f"'] & B \end{tikzcd}$ |
| `\xymatrix {A \ar @/_5pc/[r]^f &B}` | $\begin{tikzcd} A \arrow[r, bend right=80, "f"'] & B \end{tikzcd}$ |
| `\xymatrix {A \ar @{=>}@/^/[r]^f &B}` | $\begin{tikzcd} A \arrow[r, bend left, Rightarrow, "f"] & B \end{tikzcd}$ |
| `\xymatrix {A \ar @{<./}@/_/[r]_f &B}` | $\begin{tikzcd} A \arrow[r, bend right, dotted, <-\|, "f"'] & B \end{tikzcd}$ |

# Arrows' Heads

It is possible to select arrows' heads (tips) from certain fonts. Simply add
`\SelectTips{tip's family}{tip's size}`
just before the `\xymatrix` command. Here is a table showing the possible tip's families and sizes:

| **Family** | **10** | **11** | **12** |
|---|---|---|---|
| xy fonts | $\longrightarrow$ | $\longrightarrow$ | $\longrightarrow$ |
| computer modern fonts | $\longrightarrow$ | $\longrightarrow$ | $\longrightarrow$ |
| Euler math fonts | $\longrightarrow$ | $\longrightarrow$ | $\longrightarrow$ |

For example, the arrow in

```
\xymatrix{A\ar[r] &B}
```

$A \longrightarrow B$

has a head that comes by default, but using Euler math font size 12, we get:

```
\SelectTips{eu}{12}\xymatrix{A\ar[r] &B}
```

$A \longrightarrow B$

Once a selection is made, it has an affect on all of the commutative diagrams that follow. To stop the affect, use the command `\NoTips` at the place, in your document, where you want to deactivate the selection and go back using the default setting.

Here is a second example on selecting heads:

```
\xymatrix@1{
      A \ar[r] |-{\SelectTips{cm}{}\object@{>>}}
               |>{\SelectTips{eu}{}\object@{>}}
      & B }
```

$A \twoheadrightarrow\!\!\rightarrow B$

The type of arrow's head can be selected for an entire LaTeX document by the declaration
`\SelectTips{tip's family}{tip's size}` in the document preamble.

## 1.8 Cubes and More!

The cube offers a good example of how to typeset a commutative diagram. The following cube has Euler-math-font-size-12 arrow's heads:

$$
\begin{tikzcd}
A \arrow[dd] \arrow[rd] \arrow[rr] & & B \arrow[dd] \arrow[rd] & \\
& A' \arrow[rr, crossing over] & & B' \arrow[dd] \\
C \arrow[rr] \arrow[rd] & & D \arrow[rd] & \\
& C' \arrow[rr] \arrow[from=uu, crossing over] & & D'
\end{tikzcd}
$$

It is produced by the code:

```
\SelectTips{eu}{12}
\[ \xymatrix{ A  \ar[dd] \ar[rd] \ar[rr] && B \ar'[d][dd] \ar[rd] \\
            & A' \ar[dd]  \ar[rr]  && B' \ar[dd] \\
            C \ar'[r][rr]  \ar[rd] && D \ar[rd] \\
            & C' \ar[rr] && D' }
\]
```

Notice the use of `'[d]` and `'[r]` to make holes in the arrows.

The following prism has xy-font-size-12 arrow's heads:

$$
\begin{tikzcd}
A \arrow[rrr] \arrow[rrd] \arrow[dd] & & & C \arrow[dd] \\
& & B \arrow[ru] & \\
A' \arrow[rrr] \arrow[rrd] & & & C' \\
& & B' \arrow[ru] \arrow[from=uu, crossing over] &
\end{tikzcd}
$$

It is produced by the code:

```
\SelectTips{xy}{12}
\[ \xymatrix{ A \ar[rrr] \ar[rrd] \ar[dd] &&& C \ar[dd] \\
            && B \ar[ru] \ar[dd] \\
            A' \ar'[rr][rrr]  \ar[rrd] &&& C' \\
            && B' \ar[ru] }
\]
```

The following cube has Computer-modern-font-size-12 arrow's heads:

$$\begin{tikzcd}
 & \Sigma^L \arrow[rrrr] \arrow[ddd] & & & & \Sigma^R \arrow[ddd] \\
L \arrow[ru] & & L_r \arrow[ll] \arrow[rr] & & R \arrow[ru] & \\
L_m \arrow[u] \arrow[dd] & & K_{r,m} \arrow[u] \arrow[rr] \arrow[dd] \arrow[ll] & & R_{m^*} \arrow[u] \arrow[dd] & \\
 & \Sigma^G \arrow[rrrr] & & & & \Sigma^H \\
G \arrow[ru] & & G_{r^*} \arrow[rr] \arrow[ll] & & H \arrow[ru, dashed] &
\end{tikzcd}$$

It is produced by the code:

```
\SelectTips{cm}{12}
\[
\xymatrix@!=3pc{
 & \Sigma^L \ar[rrrr]\ar[ddd]&                & & &  \Sigma^R\ar[ddd]\\
 L\ar[ru]          & &   L_r\ar'[l][ll]\ar[rr]  & &  R\ar[ru]\\
 L_m\ar[u]\ar[dd]  & & K_{r,m}\ar[u]\ar[rr]\ar[dd]\ar'[l][ll] &&
 R_{m^*}\ar[u]\ar[dd]\\
 &  \Sigma^G\ar'[r]'[rrr][rrrr]             & & & &  \Sigma^H\\
 G\ar[ru]          & &   G_{r^*}\ar[rr]\ar[ll]  & &  H\ar@{-->}[ru]
}
\]
```

Notice the use of `'[l]` and `'[r]` and `'[rrr]` to make holes in the arrows.

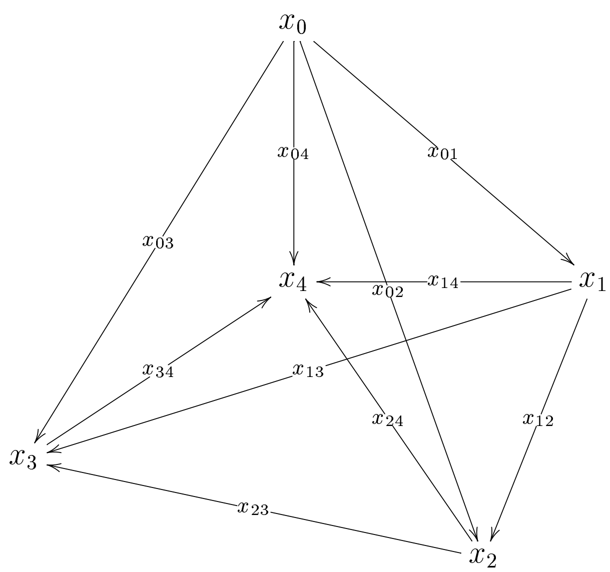
```
% ‘‘2 cubes’’ matrix.
% Adapted from source by T. Scavo, 1994-7-18.

$$\xy\xymatrix{
%
  &  &  [-1,1] \ar[rrr]^G \ar[dl]_{L_2}
        &  &  & [-1,1] \ar[dl]^{L_2} \\
  &  [0,1] \ar[rrr]^{F_4}
     &  &  & [0,1] \\
  [0,1] \ar[rrr]^D \ar[ur]^S \ar[ddd]_T
  &  &  & [0,1] \ar[ur]_S \ar[ddd]^T \\
  &  &  S^1 \ar’[r]’[rr][rrr] \ar@{}[rrr]^D
           \ar[dl]_{2C}
           \ar’[u]’[uu][uuu] \ar@{}[uuu]^C
        &  &  & S^1 \ar[uuu]_C \ar[dl]^{2C} \\
  &  [-2,2] \ar’[rr][rrr] \ar@{}[rrr]^{Q_{-2}}
           \ar’[uu][uuu] \ar@{}[uuu]^{L_1}
     &  &  & [-2,2] \ar[uuu]_{L_1} \\
  [0,1] \ar[rrr]^T \ar[ur]^U
  &  &  &  [0,1] \ar[ur]_U
%
}\endxy$$
```

$$
\begin{tikzcd}
 & & {[-1,1]} \arrow[rrr, "G"] \arrow[dl, "L_2"'] & & & {[-1,1]} \arrow[dl, "L_2"] \\
 & {[0,1]} \arrow[rrr, "F_4"] & & & {[0,1]} & \\
{[0,1]} \arrow[rrr, "D"] \arrow[ur, "S"] \arrow[ddd, "T"'] & & & {[0,1]} \arrow[ur, "S"'] \arrow[ddd, "T"] & & \\
 & & S^1 \arrow[rrr, "D"] \arrow[dl, "2C"'] \arrow[uuu, "C"] & & & S^1 \arrow[uuu, "C"'] \arrow[dl, "2C"] \\
 & {[-2,2]} \arrow[rrr, "Q_{-2}"] \arrow[uuu, "L_1"] & & & {[-2,2]} \arrow[uuu, "L_1"'] & \\
{[0,1]} \arrow[rrr, "T"] \arrow[ur, "U"] & & & {[0,1]} \arrow[ur, "U"'] & &
\end{tikzcd}
$$

```
%  ``4-Simplex Model''
%  Adapted from source by John Duskin, May 2001.

\[
\begin{xymatrix} { &&& x_0 \ar[ddd]|{x_{04}} \ar[dddddlll]|{x_{03}}
\ar[ddddddrr]|{x_{02}} \ar[dddrrr]|{x_{01}} & & &\\ & & & & & & &\\ &
& & & & & &\\
  & & & x_4    & & & x_1 \ar[dddl]|{x_{12}} \ar[ddllllll]|{x_{13}}
  \ar[lll]|{x_{14}}   \\ & & &  &  & & & \\ x_3 \ar[rrruu]|{x_{34}} &
  &  &  &  &  & \\
 & & & & & x_2 \ar[lllllu]|{x_{23}} \ar[lluuu]|{ x_{24}} & }
\end{xymatrix}
\]
```



## 1.9 Application to adjunction

Here is an application of `\xymatrix` to displaying “adjoint arrows.” In the code below, the option `@{-}` is to produce an arrow without a head or tail. The option `@*{[|<1pt>]}` is to increase the thickness of the horizontal line “arrow” by 1 point; it requires loading the package (dvips).

```
\[
\xymatrix @R=.6em{
&C \ar[rr] && B^A & **[r]\textrm{of A}\\
\ar@*{[|<1pt>]}@{-}[rrrr] &&&&     \\
&C\times A   \ar[rr] && B & **[r]\textrm{of A}\\
\ar@*{[|<1pt>]}@{-}[rrrr] &&&&     \\
&A \ar[rr] && B^C & **[r]\textrm{of A}\\
\ar@*{[|<1pt>]}@{-}[rrrr] &&&&     \\
&B^C \ar[rr] && A & **[r]\textrm{of  $\textrm{A}^{\textrm{op}}$}}
\]
```

$$\begin{array}{cl} C \longrightarrow B^A & \text{of A} \\ \hline C \times A \longrightarrow B & \text{of A} \\ \hline A \longrightarrow B^C & \text{of A} \\ \hline B^C \longrightarrow A & \text{of A}^{\text{op}} \end{array}$$

<u>Side Note:</u> You may color your arrows or lines, if you wish, by using the option `@*{[color]}`, where color can be blue, green, red, yellow, etc. Here is an example, but you will not see the affect in this uncolored document:

```
\[
\xymatrix{ A\ar@*{[green]}@*{[|<5pt>]}[rrr] &&& B}
\]
```

$$A \longrightarrow B$$

# Chapter 2

# Examples on Commutative Diagrams

◊

When you
have finished
studying all the previous
examples, then you will be able
to analyze the following collection of
codes of commutative diagrams. They are
presented in the hope that they will
be useful. Several of these need
to be refined, which is
something I hope
to do.

◊

**Objective** and **Input**

*The commands to produce the diagrams displayed on the right are as follows, respectively:*

**Output**

(*what you get*)

```
\xymatrix@1{U & S\ar[r]\ar[l] &V}
```

$$U \longleftarrow S \longrightarrow V$$

<u>Note:</u> The `@1`, in the code above, is a special code that can be used for "one-line" diagrams to improve the placement on the line.

```
\xymatrix{
\mathsf{C}/{S_0} \ar@<-.7ex>[r]_-{p_!} &
\mathsf{C}/S\ar@<-.7ex>[l]_-{p^*}}
```

$$\mathsf{C}/S_0 \underset{p_!}{\overset{p^*}{\leftrightarrows}} \mathsf{C}/S$$

```
\xymatrix{
      & A \ar[r]^f \ar[d]|{i_A} \ar[dl]_{g^{-1}}
       & B  \ar[dl]^{f^{-1}} \\
        C \ar[r]_g & A }
```

$$\begin{tikzcd}
 & A \arrow[r, "f"] \arrow[d, "i_A" description] \arrow[dl, "g^{-1}"'] & B \arrow[dl, "f^{-1}"] \\
C \arrow[r, "g"'] & A &
\end{tikzcd}$$

```
{
\xymatrixcolsep{2.5pc}\xymatrixrowsep{3pc}
\xymatrix{
0\ar[r]  &S\otimes N\ar[r]  &S\otimes M
\ar@<.7ex>[r]^-{1\otimes \varepsilon_2}
\ar@<-.7ex>[r]_-{1\otimes g\varepsilon_1} \ar[d]^g &
**[r]S\otimes M\otimes S\ar[d]_{g_3}\\
0\ar[r]  &M\ar[r]  &M\otimes S
\ar@<.7ex>[r]^-{\varepsilon_2}
\ar@<-.7ex>[r]_-{\varepsilon_2} &  **[r]M\otimes S\otimes S}
}
```

$$\begin{tikzcd}
0 \arrow[r] & S\otimes N \arrow[r] & S\otimes M \arrow[r, shift left, "1\otimes\varepsilon_2"] \arrow[r, shift right, "1\otimes g\varepsilon_1"'] \arrow[d, "g"] & S\otimes M\otimes S \arrow[d, "g_3"'] \\
0 \arrow[r] & M \arrow[r] & M\otimes S \arrow[r, shift left, "\varepsilon_2"] \arrow[r, shift right, "\varepsilon_2"'] & M\otimes S\otimes S
\end{tikzcd}$$

```
\xymatrix@!=2.5pc{
X_{d_1(\theta)}
\ar[rr]^-{\gamma_\theta}\ar@{^(->}[d] &&
X_{d_0(\theta)}\ar@{^(->}[d]\\
\coprod_{\theta\in {\mathcal C}_1}X_{d_1\theta}\approxeq
{\mathcal C}_1\times_{{\mathcal C}_0}X
\ar[rr]^-\gamma \ar[d]_{pr_1=d^*_1x}
&&X\ar[d]^x\\
{\mathcal C}_1 \ar[rr]^{d_0} &&{\mathcal C}_0}
```

$$\begin{tikzcd}
X_{d_1(\theta)} \arrow[rr, "\gamma_\theta"] \arrow[d, hook] & & X_{d_0(\theta)} \arrow[d, hook] \\
\coprod_{\theta\in\mathcal{C}_1} X_{d_1\theta} \approxeq \mathcal{C}_1\times_{\mathcal{C}_0} X \arrow[rr, "\gamma"] \arrow[d, "pr_1=d_1^*x"'] & & X \arrow[d, "x"] \\
\mathcal{C}_1 \arrow[rr, "d_0"] & & \mathcal{C}_0
\end{tikzcd}$$

```
\xymatrix@!=2.9pc{
&\left({\mathcal F}_{{\mathcal C}_0}\right)^\tau
\ar[dr]^{U^\tau}\ar@<.7ex>[dl]^{U^\tau} &\\
{\mathcal F}_{{\mathcal C}_0}\ar@<.7ex>[ur]^{F^\tau}
\ar@<-.7ex>[rr]_T &&
{\mathcal F}_{{\mathcal C}_0}\ar@<-.7ex>[ll]_S}
```

$$\begin{tikzcd}
& \left(\mathcal{F}_{\mathcal{C}_0}\right)^\tau \arrow[dr, "U^\tau"] \arrow[dl, shift left, "U^\tau"] & \\
\mathcal{F}_{\mathcal{C}_0} \arrow[ur, shift left, "F^\tau"] \arrow[rr, shift right, "T"'] & & \mathcal{F}_{\mathcal{C}_0} \arrow[ll, shift right, "S"']
\end{tikzcd}$$

```
{
\xymatrixrowsep{2pc}\xymatrixcolsep{2.5pc}
\xymatrix{  1_{S_0}\circ T\ar[r]^{\eta * 1_T}\ar[dr]_\approxeeq &
T\circ T\ar[d]^\mu &
T\circ 1_{S_0}\ar[l]_{1_T * \eta}\ar[dl]^\approxeq\\
& T &}
}
```

$$\begin{tikzcd}
1_{S_0}\circ T \arrow[r, "\eta * 1_T"] \arrow[dr, "\approxeq"'] & T\circ T \arrow[d, "\mu"] & T\circ 1_{S_0} \arrow[l, "1_T * \eta"'] \arrow[dl, "\approxeq"] \\
& T &
\end{tikzcd}$$

```
\xymatrix{
**[l] (N\otimes S)\otimes S\approxeq S\otimes (N\otimes S)
\ar[r]^{1\otimes\eta}\ar[dr]_{\eta\otimes 1} & S\otimes M\ar[d]^g\\
& M\otimes S}
```

$$\begin{tikzcd}
(N\otimes S)\otimes S\approxeq S\otimes (N\otimes S) \arrow[r, "1\otimes\eta"] \arrow[dr, "\eta\otimes 1"'] & S\otimes M \arrow[d, "g"] \\
& M\otimes S
\end{tikzcd}$$

```
\xymatrixrowsep{3.3pc}\xymatrixcolsep{4pc}
\xymatrix{
d^*_2d^*_1X\approxeq d^*_1d^*_1X
\ar@<.7ex>[rr]^{c(d_2,d^*_1X)}\ar@<-.7ex>[rr]_{c(d_1,d^*_1X)}
\ar[dd]_{d^*_1\theta\; \hskip .4in{\txt{\tt(COC)}}\hskip -.8in}
\ar[dr]^{d^*_2\theta} &&
d^*_1X \ar@<.7ex>[r]^{c(d_1,X)}\ar[dd]^\theta &
X\ar@<.7ex>[l]^{c(e,d^*_1X)}\\
& d^*_2d^*_0X\approxeq  d^*_0d^*_1X
\ar[ur]^{c(d_0,d^*_1X)} \ar[dr]_{c(d_2,d^*_0X)}
\ar[dl]^{d^*_0\theta} &&\\
d^*_0d^*_0X\approxeq  d^*_1d^*_0X
\ar@<.7ex>[rr]^{c(d_1,d^*_0X)}\ar@<-.7ex>[rr]_{c(d_0,d^*_0X)}&&
d^*_0X \ar@<.7ex>[r]^{c(d_0,X)}& X\ar@<.7ex>[l]^{c(e,d^*_0X)}}
```

$$\begin{tikzcd}
d_2^*d_1^*X \approxeq d_1^*d_1^*X \arrow[rr, shift left, "c(d_2,d_1^*X)"] \arrow[rr, shift right, "c(d_1,d_1^*X)"'] \arrow[dd, "d_1^*\theta"'] \arrow[dr, "d_2^*\theta"] \arrow[ddr, phantom, "\texttt{(COC)}"] & & d_1^*X \arrow[r, shift left, "c(d_1,X)"] \arrow[dd, "\theta"] & X \arrow[l, shift left, "c(e,d_1^*X)"] \\
& d_2^*d_0^*X \approxeq d_0^*d_1^*X \arrow[ur, "c(d_0,d_1^*X)"] \arrow[dr, "c(d_2,d_0^*X)"'] \arrow[dl, "d_0^*\theta"] & & \\
d_0^*d_0^*X \approxeq d_1^*d_0^*X \arrow[rr, shift left, "c(d_1,d_0^*X)"] \arrow[rr, shift right, "c(d_0,d_0^*X)"'] & & d_0^*X \arrow[r, shift left, "c(d_0,X)"] & X \arrow[l, shift left, "c(e,d_0^*X)"]
\end{tikzcd}$$

```
\[
\xymatrix@!R=3pc{
s\otimes m \ar[r]^-{1\otimes g\varepsilon_1}\ar[d]_g &
**[r]s\otimes g(1\otimes m ) \ar[d]^{g_3}\\
g(s\otimes m) \ar[r]^-{\varepsilon_2} &
**[r]{\begin{matrix} g_3(s\otimes g(1\otimes m))=
g_3g_1(s\otimes 1\otimes m)\hfill\\=
  g_2(s\otimes 1\otimes m)
  = g(s\otimes m)\otimes 1 =
\varepsilon_2 g(s\otimes m)\; . \end{matrix}}}
\]
```

$$
\begin{tikzcd}
s\otimes m \arrow[r, "1\otimes g\varepsilon_1"] \arrow[d, "g"'] & s\otimes g(1\otimes m) \arrow[d, "g_3"] \\
g(s\otimes m) \arrow[r, "\varepsilon_2"] & \begin{matrix} g_3(s\otimes g(1\otimes m))= g_3g_1(s\otimes 1\otimes m)\hfill\\ = g_2(s\otimes 1\otimes m) = g(s\otimes m)\otimes 1 = \varepsilon_2 g(s\otimes m)\; . \end{matrix}
\end{tikzcd}
$$

```
\[
\xymatrix@!=2.5pc{
(N\otimes N)\otimes {(S\otimes S)}
\ar@<.7ex>[rrrr]^{\mu_1}\ar@<-.7ex>[rrrr]_{\mu_2}
\ar[dr]^{1\otimes (\eta\otimes_S\eta )}
\ar[dd]_{(\eta\otimes_S\eta )\otimes 1}
&&&&    N\otimes {(S\otimes S)}
\ar[dl]_{1\otimes\eta}\ar[dd]^{\eta\otimes 1}\\
& S\otimes(M\otimes_SM)
\ar[rr]\ar[dl]^{g\otimes_Sg} && S\otimes M\ar[dr]_g &\\
(M\otimes_SM)\otimes S\ar[rrrr]
&&&&    M\otimes S}
\]
```

$$
\begin{tikzcd}
(N\otimes N)\otimes (S\otimes S) \arrow[rrr, shift left, "\mu_1"] \arrow[rrr, shift right, "\mu_2"'] \arrow[dr, "1\otimes(\eta\otimes_S\eta)"] \arrow[dd, "(\eta\otimes_S\eta)\otimes 1"'] & & & N\otimes (S\otimes S) \arrow[dl, "1\otimes\eta"'] \arrow[dd, "\eta\otimes 1"] \\
& S\otimes (M\otimes_S M) \arrow[r] \arrow[dl, "g\otimes_S g"] & S\otimes M \arrow[dr, "g"'] & \\
(M\otimes_S M)\otimes S \arrow[rrr] & & & M\otimes S
\end{tikzcd}
$$

```
\[
\xymatrixrowsep{2pc}\xymatrixcolsep{2pc}
\xymatrix{
& S_1\ar[dr]\ar[dl]&&  S_2\ar[dr]\ar[dl]&  \\
U && V && W}
\qquad\mapsto\qquad
\xymatrix@!=1pc{
&& S_1\times_VS_2 \ar[dr]\ar[dl]&& \\
&S_1\ar[dl]\ar[dr] && S_2\ar[dl]\ar[dr]&\\
U &&V &&W}\]
```

$$
\begin{tikzcd}
& S_1 \arrow[dr] \arrow[dl] & & S_2 \arrow[dr] \arrow[dl] & \\
U & & V & & W
\end{tikzcd}
\qquad\mapsto\qquad
\begin{tikzcd}
& & S_1 \times_V S_2 \arrow[dr] \arrow[dl] & & \\
& S_1 \arrow[dl] \arrow[dr] & & S_2 \arrow[dl] \arrow[dr] & \\
U & & V & & W
\end{tikzcd}
$$

## 2.1 Two Cells Diagrams

**Objective** and **Input**

*The commands to produce the diagrams displayed on the right are as follows, respectively:*

**Output**

(*what you get*)

```
\xymatrix{ A\rtwocell^f_g{\alpha} &B}
```

$$\begin{tikzcd} A \arrow[r, bend left, "f"] \arrow[r, bend right, "g"'] \arrow[r, phantom, "\Downarrow\alpha"] & B \end{tikzcd}$$

```
\xymatrix{ A\rtwocell<10>^f_g{\alpha} &B}

Notice the affect  of <10> .
```

$$\begin{tikzcd} A \arrow[r, bend left=70, "f"] \arrow[r, bend right=70, "g"'] \arrow[r, phantom, "\Downarrow\alpha"] & B \end{tikzcd}$$

```
\xymatrix{ A & B \ltwocell_f^g{\alpha}}

Notice the reversed meaning of ^ _ and => .
```

$$\begin{tikzcd} A & B \arrow[l, bend right, "f"'] \arrow[l, bend left, "g"] \arrow[l, phantom, "\alpha\Uparrow"] \end{tikzcd}$$

```
\xymatrix{ A\rtwocell^f_g{\omit} &B}
```

$$\begin{tikzcd} A \arrow[r, bend left, "f"] \arrow[r, bend right, "g"'] & B \end{tikzcd}$$

**Objective** and **Input**

*The commands to produce the diagrams displayed on the right are as follows, respectively:*

**Output**

(*what you get*)

```
\xymatrix{ A & B \ltwocell_f^g{‘\alpha}}

Notice the affect  of ‘ .
```

$$\begin{tikzcd} A \arrow[r, bend right, "g"'] \arrow[r, phantom, "\alpha"] & B \arrow[l, bend right, "f"'] \end{tikzcd}$$

```
\xymatrix{ A & B \ltwocell_f^g{’\alpha}}

Notice the affect  of ’ .
```

$$\begin{tikzcd} A \arrow[r, bend left, "f"] \arrow[r, phantom, "\alpha"] & B \arrow[l, bend left, "g"] \end{tikzcd}$$

```
\xymatrix{
A\ruppertwocell^f{\alpha}  \rlowertwocell_h{\beta} \ar[r]_(.35)g & B }
```

$$\begin{tikzcd} A \arrow[r, bend left=60, "f"] \arrow[r, "g"' near start] \arrow[r, bend right=60, "h"'] \arrow[r, phantom, shift left=2, "\Downarrow\alpha"] \arrow[r, phantom, shift right=2, "\Downarrow\beta"] & B \end{tikzcd}$$

```
\xymatrix{
& B\\
A  \urtwocell^f_g{\alpha}}
\qquad\qquad
\xymatrix{
A \ddrrtwocell\\
&&\\
&& B}
```

$$\begin{tikzcd} & B \\ A \arrow[ur, bend left, "f"] \arrow[ur, bend right, "g"'] \arrow[ur, phantom, "\Downarrow\alpha"] \end{tikzcd} \qquad\qquad \begin{tikzcd} A \arrow[ddrr, bend left] \arrow[ddrr, bend right] \arrow[ddrr, phantom, "\Downarrow"] & & \\ & & \\ & & B \end{tikzcd}$$

```
\xymatrix@!=.5pc{
& B \rrtwocell^\zeta_\xi{\gamma}&& C \ddruppertwocell^\beta{}   &\\
&&&& \\
A \uuruppertwocell^\alpha \xlowertwocell[rrrr]{}_f &&&& D }
```

$$\begin{tikzcd} & B \arrow[rr, bend left, "\zeta"] \arrow[rr, bend right, "\xi"'] \arrow[rr, phantom, "\Downarrow\gamma"] & & C \arrow[ddr, bend left, "\beta"] \arrow[ddr, phantom, "\Downarrow"] & \\ & & & & \\ A \arrow[uur, bend left, "\alpha"] \arrow[uur, phantom, "\Downarrow"] \arrow[rrrr, bend right, "f"'] \arrow[rrrr, phantom, "\Downarrow"] & & & & D \end{tikzcd}$$

```
$$\xymatrix@!C=2pc{
S_2
\ruppertwocell<5>^{p_{23}}{\omit}
\ar[r]|{p_{13}}\rlowertwocell<-5>_{p_{12}}{\omit} &S_1
\ruppertwocell<5>^{p_2}{\omit}
\rlowertwocell<-5>_{p_1}{\omit} & S_0\ar[l]|e \ar[r]|p& S}
$$
```

$$\begin{tikzcd}
S_2 \arrow[r, bend left, "p_{23}"] \arrow[r, "p_{13}" description] \arrow[r, bend right, "p_{12}"'] & S_1 \arrow[r, bend left, "p_2"] \arrow[r, bend right, "p_1"'] & S_0 \arrow[l, "e" description] \arrow[r, "p" description] & S
\end{tikzcd}$$

```
$$\xymatrix@!C=2pc{
\mathcal{F}_{S_2} &
\mathcal{F}_{S_1}\luppertwocell<-5>_{d^*_2}{\omit}
\ar[l]|{d^*_1}  \llowertwocell<5>^{d^*_0}{\omit} \ar[r]|{e^*}&
 \mathcal{F}_{S_0}\luppertwocell<-5>_{d^*_1}{\omit}
\llowertwocell<5>^{d^*_0}{\omit} }
$$
```

$$\begin{tikzcd}
\mathcal{F}_{S_2} & \mathcal{F}_{S_1} \arrow[l, bend right, "d^*_2"'] \arrow[l, "d^*_1" description] \arrow[l, bend left, "d^*_0"] \arrow[r, "e^*" description] & \mathcal{F}_{S_0} \arrow[l, bend right, "d^*_1"'] \arrow[l, bend left, "d^*_0"]
\end{tikzcd}$$

```
$$
\xymatrix@!C=2pc{
S_3\ruppertwocell<5>^{d_3}{\omit}
\ar@<1ex>[r]|{d_2}\ar@<-1ex>[r]|{d_1}
\rlowertwocell<-5>_{d_0}{\omit}&   S_2
\ruppertwocell<5>^{d_2}{\omit}
\ar[r]|{d_1}\rlowertwocell<-5>_{d_0}{\omit} &S_1
\ruppertwocell<5>^{d_1}{\omit}
\rlowertwocell<-5>_{d_0}{\omit} &S_0\ar[l]|e}
$$
```

$$\begin{tikzcd}
S_3 \arrow[r, bend left, "d_3"] \arrow[r, shift left, "d_2" description] \arrow[r, shift right, "d_1" description] \arrow[r, bend right, "d_0"'] & S_2 \arrow[r, bend left, "d_2"] \arrow[r, "d_1" description] \arrow[r, bend right, "d_0"'] & S_1 \arrow[r, bend left, "d_1"] \arrow[r, bend right, "d_0"'] & S_0 \arrow[l, "e" description]
\end{tikzcd}$$

```
$${
\xymatrixrowsep{4pc}\xymatrixcolsep{1.5pc}
\xymatrix{
& \frak{Des}(\varepsilon)\ar[dr]^U &\\
**[l]\text{Mod-}R
\rruppertwocell<5>^{\varepsilon_*}{\omit}
\rrlowertwocell<-5>_{\varepsilon_!}{\omit}
\ar[ur]^{\varepsilon^{\#}} &&
**[r]\text{Mod-}S\ar[ll]|{\varepsilon^*}}
}$$
```

$$
\begin{tikzcd}
 & \mathfrak{Des}(\varepsilon) \arrow[dr, "U"] & \\
\text{Mod-}R \arrow[ur, "\varepsilon^{\#}"] \arrow[rr, bend left, "\varepsilon_*"] \arrow[rr, bend right, "\varepsilon_!"'] & & \text{Mod-}S \arrow[ll, "\varepsilon^*" description]
\end{tikzcd}
$$

```
$$\xymatrix@!=4.5pc{
**[l]l^{S_\diamond}_{(X,\theta)} \qquad
d^*_2d^*_1X=d^*_1d^*_1X
\ruppertwocell<5>^{c(d_2,d^*_1X)}{\omit}
\ar[r]|{c(d_1,d^*_1X)}
\rlowertwocell<-5>_{c(d_0,d^*_1X)\circ d^*_2\theta}
{\omit}
\ar@<-1.3in>[d]_{P}
\ar[d]_{d^*_2d^*_1x} &
d^*_1X
\ruppertwocell<5>^{c(d_1,X)}{\omit}
\rlowertwocell<-5>_{c(d_0,X)\circ\theta}{\omit}
\ar[d]_{d^*_1x} &
X\ar[l]|{c(e,d^*_1X)} \ar[d]^x \\
**[l]S_\diamond \qquad\qquad\qquad\qquad S_2
\ruppertwocell<5>^{d_2}{\omit}
\ar[r]|{d_1}\rlowertwocell<-5>_{d_0}{\omit} &
S_1
\ruppertwocell<5>^{d_1}{\omit}
\rlowertwocell<-5>_{d_0}{\omit} &
S_0\; .\ar[l]|{e}}$$
```

$$\begin{tikzcd}[column sep=large, row sep=large]
l^{S_\diamond}_{(X,\theta)} \arrow[d, "P"'] & d^*_2d^*_1X=d^*_1d^*_1X \arrow[r, bend left, "c(d_2,d^*_1X)"] \arrow[r, "c(d_1,d^*_1X)" description] \arrow[r, bend right, "c(d_0,d^*_1X)\circ d^*_2\theta"'] \arrow[d, "d^*_2d^*_1x"'] & d^*_1X \arrow[r, bend left, "c(d_1,X)"] \arrow[r, bend right, "c(d_0,X)\circ\theta"'] \arrow[d, "d^*_1x"'] & X \arrow[l, "c(e,d^*_1X)" description] \arrow[d, "x"] \\
S_\diamond & S_2 \arrow[r, bend left, "d_2"] \arrow[r, "d_1" description] \arrow[r, bend right, "d_0"'] & S_1 \arrow[r, bend left, "d_1"] \arrow[r, bend right, "d_0"'] & S_0\; . \arrow[l, "e" description]
\end{tikzcd}$$

```
\xymatrix{
A  \ddrrtwocell<\omit>{<8>}\\
&&\\
&& B}
\qquad\qquad \xymatrix{
             A  \ddrrtwocell<\omit>{<0>}\\
             &&\\
             && B}
\qquad\qquad \xymatrix{
             A  \ddrrtwocell<\omit>{<-8>}\\
             &&\\
             && B}
```

$$\begin{tikzcd} A \arrow[ddrr, phantom, "\Swarrow"] & & \\ & & \\ & & B \end{tikzcd} \qquad\qquad \begin{tikzcd} A \arrow[ddrr, phantom, "\Swarrow"] & & \\ & & \\ & & B \end{tikzcd} \qquad\qquad \begin{tikzcd} A \arrow[ddrr, phantom, "\Swarrow"] & & \\ & & \\ & & B \end{tikzcd}$$

```
\xymatrix@!=2.5pc{
TTT \ar[r]^{T\mu }\ar[d]_{\mu T}\drtwocell<\omit>{<0>\gamma}&
TT\ar[d]^\mu \\
TT\ar[r]_\mu  &T}
\qquad\qquad
\xymatrix@!=2.5pc{
T\ar[r]^{\eta T}\ar[dr]_{Id_T}\drtwocell<\omit>{<-3>\beta} &
TT \ar[d]|\mu  &
T\ar[l]_{T\eta }\ar[dl]^{Id_T}\dltwocell<\omit>{<3>\alpha}\\
&T&}
```

$$\begin{tikzcd} TTT \arrow[r, "T\mu"] \arrow[d, "\mu T"'] \arrow[dr, phantom, "\Swarrow_\gamma"] & TT \arrow[d, "\mu"] \\ TT \arrow[r, "\mu"'] & T \end{tikzcd} \qquad\qquad \begin{tikzcd} T \arrow[r, "\eta T"] \arrow[dr, "Id_T"'] \arrow[dr, phantom, "\Swarrow_\beta"] & TT \arrow[d, "\mu" description] & T \arrow[l, "T\eta"'] \arrow[dl, "Id_T"] \arrow[dl, phantom, "\alpha\Nwarrow"] \\ & T & \end{tikzcd}$$

```
\xymatrix@!=2.5pc{
\mathcal{F}_Z
\ar[r]^{u^*} \ar[dr]_1   \drtwocell<\omit>{<-3>{\epsilon^u}}
& \mathcal{F}_X
\ar[r]^{f_!} \ar[d]_{u_!} \drtwocell<\omit>{<0>{\beta}}
& \mathcal{F}_Y
\ar[d]^{g_!} \ar[dr]^1   \drtwocell<\omit>{<3>{\eta^g}}  &\\
&  \mathcal{F}_Z
\ar[r]_{v_!} & \mathcal{F}_W \ar[r]_{g^*} & \mathcal{F}_Y\; .}
```

$$\begin{tikzcd}
\mathcal{F}_Z \arrow[r, "u^*"] \arrow[dr, "1"'] & \mathcal{F}_X \arrow[r, "f_!"] \arrow[d, "u_!"'] \arrow[l, phantom, shift right=2ex, "\Swarrow\epsilon^u"] \arrow[dr, phantom, "\Swarrow\beta"] & \mathcal{F}_Y \arrow[d, "g_!"] \arrow[dr, "1"] \arrow[dr, phantom, shift right=2ex, "\Swarrow\eta^g"] & \\
 & \mathcal{F}_Z \arrow[r, "v_!"'] & \mathcal{F}_W \arrow[r, "g^*"'] & \mathcal{F}_Y\; .
\end{tikzcd}$$

```
\xymatrix@!=1pc{
&& {\mathcal F}_{{(S_2\circ S_1)}} \ar[dr]^{(p_2)_!} &&\\
&{\mathcal F}_{S_1}
\ar[dr]_{(t_1)_!}\ar[ur]^{p^*_1}\rrtwocell<\omit>{<0>\ss} &&
{\mathcal F}_{S_2} \ar[dr]^{(t'_1)_!} &\\
{\mathcal F}_U \ar[ur]^{s^*_1} &&
{\mathcal F}_V \ar[ur]_{(s'_1)^*} &&
{\mathcal F}_W}
```

$$\begin{tikzcd}
 & & \mathcal{F}_{(S_2\circ S_1)} \arrow[dr, "(p_2)_!"] & & \\
 & \mathcal{F}_{S_1} \arrow[ur, "p_1^*"] \arrow[dr, "(t_1)_!"'] \arrow[rr, phantom, "\Downarrow ß"] & & \mathcal{F}_{S_2} \arrow[dr, "(t'_1)_!"] & \\
\mathcal{F}_U \arrow[ur, "s_1^*"] & & \mathcal{F}_V \arrow[ur, "(s'_1)^*"'] & & \mathcal{F}_W
\end{tikzcd}$$

```
$$\xymatrix@!=2.3pc{
&& {\mathcal F}_{{\mathcal C}_1}\ar[ddrr]^{(d_0)_!} &&\\
&& {\mathcal F}_{{\mathcal C}_2}\ar[dr]^{(d_0)_!}   &&\\
{\mathcal F}_{{\mathcal C}_0}\ar[uurr]^{d^*_1}\ar[r]^{d^*_1}&
{\mathcal F}_{{\mathcal C}_1}\ar[ur]^{d^*_2}\ar[dr]_{(d_0)_!}&&
{\mathcal F}_{{\mathcal C}_1}\ar[r]^{(d_0)_!}
\lltwocell<\omit>{<0>\ss^{-1}\;\;}
\lltwocell<\omit>{<15>\tilde{d_1}} &
{\mathcal F}_{{\mathcal C}_0}\\
&& {\mathcal F}_{{\mathcal C}_0}\/.\ar[ur]_{d^*_1}&&}$$
```

$$\begin{tikzcd}
 & & \mathcal{F}_{\mathcal{C}_1} \arrow[ddrr, "(d_0)_!"] \arrow[d, phantom, "\tilde{d_1}\Uparrow"] & & \\
 & & \mathcal{F}_{\mathcal{C}_2} \arrow[dr, "(d_0)_!"] \arrow[dd, phantom, "ß^{-1}\Uparrow"] & & \\
\mathcal{F}_{\mathcal{C}_0} \arrow[uurr, "d_1^*"] \arrow[r, "d_1^*"] & \mathcal{F}_{\mathcal{C}_1} \arrow[ur, "d_2^*"] \arrow[dr, "(d_0)_!"'] & & \mathcal{F}_{\mathcal{C}_1} \arrow[r, "(d_0)_!"] & \mathcal{F}_{\mathcal{C}_0} \\
 & & \mathcal{F}_{\mathcal{C}_0}. \arrow[ur, "d_1^*"'] & &
\end{tikzcd}$$

```
$$\hskip -.3truein\xymatrix{
TTTT
\ddrrtwocell<\omit>{<-5>\;\; T\gamma}\ddrtwocell<\omit>{<2>\;\gamma T}
\ar[rr]^{TT\mu } \ar[dr]^{T\mu T} \ar[dd]_{\mu TT}&&
TTT\ar[dr]^{T\mu } &\\
&TTT
\ddrrtwocell<\omit>{<0>\gamma}\ar[rr]_{T\mu }\ar[dd]^{\mu T} &&
TT\ar[dd]^\mu \\
TTT\ar[dr]_{\mu T}&&\\
&TT\ar[rr]_{\mu } && T}
\qquad =\qquad
\xymatrix{
TTTT
\ddrrtwocell<\omit>{<0>\;\;\; \mu^{-1}_\mu}\ar[rr]^{TT\mu }
\ar[dd]_{\mu TT}&& TTT\ddrrtwocell<\omit>{<5>\gamma}
\ar[dd]^{\mu T}\ar[dr]^{T\mu } &\\
&&&TT\ar[dd]^\mu \\
TTT\ddrrtwocell<\omit>{<-5>\gamma}\ar[dr]_{\mu T} \ar[rr]^{T\mu }  &&
TT\ar[dr]^\mu  &&\\
&TT \ar[rr]_\mu  && T\\
&&&&}$$
```

$$
\begin{tikzcd}
TTTT \arrow[rr, "TT\mu"] \arrow[dr, "T\mu T"] \arrow[dd, swap, "\mu TT"] \arrow[drr, phantom, "\Swarrow T\gamma"] \arrow[d, phantom, "\Swarrow \gamma T"] & & TTT \arrow[dr, "T\mu"] & \\
& TTT \arrow[rr, swap, "T\mu"] \arrow[dd, "\mu T"] \arrow[ddrr, phantom, "\Swarrow \gamma"] & & TT \arrow[dd, "\mu"] \\
TTT \arrow[dr, swap, "\mu T"] & & & \\
& TT \arrow[rr, swap, "\mu"] & & T
\end{tikzcd}
\qquad = \qquad
\begin{tikzcd}
TTTT \arrow[rr, "TT\mu"] \arrow[dd, swap, "\mu TT"] \arrow[ddrr, phantom, "\Swarrow \mu_\mu^{-1}"] & & TTT \arrow[dd, "\mu T"] \arrow[dr, "T\mu"] \arrow[ddrr, phantom, "\Swarrow \gamma"] & \\
& & & TT \arrow[dd, "\mu"] \\
TTT \arrow[dr, swap, "\mu T"] \arrow[rr, "T\mu"] \arrow[drr, phantom, "\Swarrow \gamma"] & & TT \arrow[dr, "\mu"] & \\
& TT \arrow[rr, swap, "\mu"] & & T
\end{tikzcd}
$$

```
$$\xymatrix@!=1pc{
&&TT\ar[dr]^\mu &\\
**[l]TT\ar[r]^-{T\eta T} &
TTT\rrtwocell<\omit>{<0>\gamma}\ar[ur]^{T\mu }
\ar[dr]_{\mu T}&&T\\
&&TT\ar[ur]_\mu &}
\qquad =\qquad
\xymatrix@!=1pc{
&TTT\ar[dr]^{T\mu }&&\\
TT\ar[rr] \rrtwocell<\omit>{<-2>{\;\;T\beta}}
\rrtwocell<\omit>{<2>{\;\alpha T}}
\ar[ur]^{T\eta T}\ar[dr]_{T\eta T}&&
TT\ar[r]^\mu  &**[r]T \\
&TTT\ar[ur]_{\mu T}&&}$$
```

$$\begin{tikzcd}
 & & TT \arrow[dr, "\mu"] & \\
TT \arrow[r, "T\eta T"] & TTT \arrow[ur, "T\mu"] \arrow[dr, "\mu T"'] \arrow[rr, phantom, "\Downarrow\gamma"] & & T \\
 & & TT \arrow[ur, "\mu"'] &
\end{tikzcd}
\qquad = \qquad
\begin{tikzcd}
 & TTT \arrow[dr, "T\mu"] & & \\
TT \arrow[ur, "T\eta T"] \arrow[dr, "T\eta T"'] \arrow[rr] \arrow[rr, phantom, shift left=3ex, "\Downarrow T\beta"] \arrow[rr, phantom, shift right=3ex, "\Downarrow \alpha T"] & & TT \arrow[r, "\mu"] & T \\
 & TTT \arrow[ur, "\mu T"'] & &
\end{tikzcd}$$

```
$$\xymatrix@!=1pc{
&&&TB\ar[dr]^b &\\
A \ar[rr]^-{\eta_A}\ar[drrr]_{1_A}& \drrtwocell<\omit>{<0>\psi}&
TA\rrtwocell<\omit>{<0>\rho}\ar[ur]^{Tf }
\ar[dr]^{a}&&B\\
&&&A\ar[ur]_f &}
\qquad =\qquad
\xymatrix@!=1pc{
&TA\ar[dr]^{Tf }&&\\
A \rrtwocell<\omit>{<0>{\;\;\eta_f}}
\ar[ur]^{\eta_A}\ar[dr]_{f}&&TB\ar[rr]^b  && B \\
&B\urrtwocell<\omit>{<0>\Psi}\ar[ur]^{\eta_B}\ar[rrru]_{1_B}&&}$$
```

$$
\begin{tikzcd}
& & & TB \arrow[dr, "b"] & \\
A \arrow[rr, "\eta_A"] \arrow[drrr, "1_A"'] & {} \arrow[drr, phantom, "\Downarrow\psi"] & TA \arrow[ur, "Tf"] \arrow[dr, "a"] \arrow[rr, phantom, "\Downarrow\rho"] & & B \\
& & & A \arrow[ur, "f"'] &
\end{tikzcd}
\qquad = \qquad
\begin{tikzcd}
& TA \arrow[dr, "Tf"] & & & \\
A \arrow[ur, "\eta_A"] \arrow[dr, "f"'] \arrow[rr, phantom, "\Downarrow\eta_f"] & & TB \arrow[rr, "b"] & & B \\
& B \arrow[ur, "\eta_B"'] \arrow[rrru, "1_B"'] \arrow[urr, phantom, "\Downarrow\Psi"] & & &
\end{tikzcd}
$$

```
$\xymatrixcolsep{5pc}\xymatrixrowsep{5pc}
\forall\qquad \xymatrix{
A\rtwocell^f_{f'}{\alpha} &A'}$,\;  the following equation holds

$$\xymatrixcolsep{5pc}\xymatrixrowsep{5pc}
\xymatrix{
A \rtwocell^{f}_{f'}{\quad \alpha}\ar[d]_{\eta_A}&
A' \ar[d]^{\eta_{A'}}\\
TA\ar[r]^{Tf} &TA'\ultwocell<\omit>{<0>{\eta_f}}}
\qquad = \qquad
\xymatrix{
TA\ar[r]^{Tf'} \ar[d]_{\eta_A} &A'\ar[d]^{\eta_{A'}}\\
TA \rtwocell^{Tf}_{Tf'}{\quad T\alpha}& TA'
\ultwocell<\omit>{<0>{\eta_{f'}}}}$$
```

$\forall \qquad$ $\begin{tikzcd} A \arrow[r, bend left, "f"] \arrow[r, bend right, "f'"'] \arrow[r, phantom, "\Downarrow\alpha"] & A' \end{tikzcd}$ , the following equation holds

$$\begin{tikzcd}[column sep=5pc, row sep=5pc] A \arrow[r, bend left, "f"] \arrow[r, bend right, "f'"'] \arrow[r, phantom, "\Downarrow\ \alpha"] \arrow[d, "\eta_A"'] & A' \arrow[d, "\eta_{A'}"] \\ TA \arrow[r, "Tf"] \arrow[ur, phantom, "\eta_f \Nearrow"] & TA' \end{tikzcd} \qquad = \qquad \begin{tikzcd}[column sep=5pc, row sep=5pc] TA \arrow[r, "Tf'"] \arrow[d, "\eta_A"'] & A' \arrow[d, "\eta_{A'}"] \\ TA \arrow[r, bend left, "Tf"] \arrow[r, bend right, "Tf'"'] \arrow[r, phantom, "\Downarrow T\alpha"] \arrow[ur, phantom, "\eta_{f'} \Nearrow"] & TA' \end{tikzcd}$$

```
$${
\xymatrixrowsep{4pc}\xymatrixcolsep{6pc}
\xymatrix{
d^*_1X \ar[r]^a_\approxeq
\dtwocell<5.5>_{d^*_1m\quad }^{\quad d^*_1n}
{\omit {\stackrel{d^*_1\zeta}\Longrightarrow}} &
d^*_0X\ar[d]^{d^*_0n}\\
d^*_1Y\ar[r]^\approxeq_b & d^*_0Y  \ultwocell<\omit>{\rho}}
\qquad =\qquad
\xymatrix{
d^*_1X \ar[r]^a_\approxeq \ar[d]_{d^*_1m} &
d^*_0X\dtwocell<5.5>_{d^*_0m\quad }^{\quad d^*_0n}
{\omit {\stackrel{d^*_0\zeta}\Longrightarrow}}\\
d^*_1Y \ar[r]^\approxeq_b  & d^*_0Y  \ultwocell<\omit>{\eta}}
}$$
```

$$
\begin{tikzcd}[row sep=4pc, column sep=6pc]
d_1^*X \arrow[r, "a", "\approxeq"'] \arrow[d, bend right, "d_1^*m"'] \arrow[d, bend left, "d_1^*n"] \arrow[d, phantom, "\overset{d_1^*\zeta}{\Longrightarrow}"] & d_0^*X \arrow[d, "d_0^*n"] \\
d_1^*Y \arrow[r, "\approxeq", "b"'] \arrow[ur, phantom, "{}^{\rho}\!\Nearrow"] & d_0^*Y
\end{tikzcd}
\qquad = \qquad
\begin{tikzcd}[row sep=4pc, column sep=6pc]
d_1^*X \arrow[r, "a", "\approxeq"'] \arrow[d, "d_1^*m"'] & d_0^*X \arrow[d, bend right, "d_0^*m"'] \arrow[d, bend left, "d_0^*n"] \arrow[d, phantom, "\overset{d_0^*\zeta}{\Longrightarrow}"] \\
d_1^*Y \arrow[r, "\approxeq", "b"'] \arrow[ur, phantom, "{}^{\eta}\!\Nearrow"] & d_0^*Y
\end{tikzcd}
$$

```
$${
\xymatrixrowsep{6pc}\xymatrixcolsep{6pc}
\xymatrix{
{}\ar[r]^{d^*_2d^*_1a=d^*_1d^*_1a}\ar[d]_{d^*_3d^*_2a}
\ar[dr]|{d^*_1d^*_2a=d^*_2d^*_1a}& {}
\ar@{}[dl]|(.23){\big\Downarrow d^*_1\alpha}
           |(.75){\big\Downarrow d^*_3\alpha}\\
{}\ar[r]_{d^*_3d^*_0a}& {}\ar[u]_{d^*_1d^*_0a}}
\qquad =\qquad
\xymatrix{
{}\ar[r]^{d^*_2d^*_1a=d^*_1d^*_1a}\ar[d]_{d^*_3d^*_2a=d^*_2d^*_2a}
\ar@{}[dr]|(.23){\big\Downarrow d^*_2\alpha}
           |(.75){\big\Downarrow d^*_0\alpha} &{}\\
{}\ar[r]_{d^*_0d^*_2a}
\ar[ur]|{d^*_2d^*_0a=d^*_0d^*_1a}&{}\ar[u]_{d^*_0d^*_0a}}
}$$
```

$$
\begin{tikzcd}[row sep=6pc, column sep=6pc]
{} \arrow[r, "d_2^*d_1^*a=d_1^*d_1^*a"] \arrow[d, swap, "d_3^*d_2^*a"] \arrow[dr, "d_1^*d_2^*a=d_2^*d_1^*a" description] & {} \arrow[dl, phantom, "\Downarrow d_1^*\alpha" near start, "\Downarrow d_3^*\alpha" near end] \\
{} \arrow[r, swap, "d_3^*d_0^*a"] & {} \arrow[u, swap, "d_1^*d_0^*a"]
\end{tikzcd}
\qquad = \qquad
\begin{tikzcd}[row sep=6pc, column sep=6pc]
{} \arrow[r, "d_2^*d_1^*a=d_1^*d_1^*a"] \arrow[d, swap, "d_3^*d_2^*a=d_2^*d_2^*a"] \arrow[dr, phantom, "\Downarrow d_2^*\alpha" near start, "\Downarrow d_0^*\alpha" near end] & {} \\
{} \arrow[r, swap, "d_0^*d_2^*a"] \arrow[ur, "d_2^*d_0^*a=d_0^*d_1^*a" description] & {} \arrow[u, swap, "d_0^*d_0^*a"]
\end{tikzcd}
$$

## 2.2 Curved Arrows

**Warning:** Using curves can be a quite a strain on TEX's memory; you should therefore limit the length and number of curves used on a single page. You may use `\vfill\eject` at certain points.

```
\xymatrix@!=2.3pc{
      S_1 \ar@/_/[ddr]_1 \ar@/^/[drr]^{e\circ d_0}
        \ar@{.>}[dr]|-{k_0}            \\
       & S_2
\ar[d]^{d_0} \ar[r]_{d_1}
                      & S_1 \ar[d]_{d_0}       \\
       & S_1 \ar[r]^{d_0}   & S_0                }
```

$$\begin{tikzcd}
S_1 \arrow[ddr, bend right, "1"'] \arrow[drr, bend left, "e\circ d_0"] \arrow[dr, dotted, "k_0" description] & & \\
& S_2 \arrow[d, "d_0"] \arrow[r, "d_1"'] & S_1 \arrow[d, "d_0"'] \\
& S_1 \arrow[r, "d_0"] & S_0
\end{tikzcd}$$

```
\xymatrix@1{
\mathcal{F}_{S_0} & \mathfrak{Des}_\mathcal{F} \ar[l]_-{\txt{I}} &
\mathcal{F}_S \ar[l]_-{d_0^\#}  \ar@/^/@<+1ex>[ll]^{d^*_0}}
```

$$\begin{tikzcd}
\mathcal{F}_{S_0} & \mathfrak{Des}_\mathcal{F} \arrow[l, "\mathrm{I}"'] & \mathcal{F}_S \arrow[l, "d_0^\#"'] \arrow[ll, bend left, "d_0^*"]
\end{tikzcd}$$

```
\xymatrix{
A \ar[r] ^a="a" & B \ar[r] ^b="b" & C  \ar @/^/ "a";"b" }
```

$$\begin{tikzcd}
A \arrow[r, "a"{name=a}] & B \arrow[r, "b"{name=b}] & C \arrow[from=a, to=b, bend left]
\end{tikzcd}$$

```
\xymatrix@!=2pc{
&&  {\mathcal F}_{{(S_1\times_V S_2)}} \ar[dr]^{(p_2)_!}\relax
\ar@/^2pc/@{:>}[dddd]|(.25){\widetilde{g*f}}  &&\\
& {\mathcal F}_{S_1}\ar[ur]^{p^*_1}\ar[dr]_{(t_1)_!} &&
{\mathcal F}_{S_2} \ar[dr]^{(t'_1)_!} &\\
{\mathcal F}_U
\rrtwocell<\omit>{<0>{\tilde f}}\ar[ur]^{s^*_1}\ar[dr]_{s^*_2}
&& {\mathcal F}_V
\rrtwocell<\omit>{<0>{\tilde g}}\ar[ur]_{(s'_1)^*}
\ar[dr]_{(s'_2)^*}&&{\mathcal F}_W\\
& {\mathcal F}_{T_1}\ar[dr]_{\pi^*_1}\ar[ur]_{(t_2)_!} &&
{\mathcal F}_{T_2}\ar[ur]_{(t'_2)_!} &\\
&& \mathcal{F}_{(T_1\times_V T_2)}\ar[ur]_{(\pi_2)_!} &&}
```

$$\begin{tikzcd}
 & & \mathcal{F}_{(S_1\times_V S_2)} \arrow[dr, "(p_2)_!"] \arrow[dddd, bend left, dotted, "\widetilde{g*f}"] & & \\
 & \mathcal{F}_{S_1} \arrow[ur, "p_1^*"] \arrow[dr, "(t_1)_!"'] & & \mathcal{F}_{S_2} \arrow[dr, "(t'_1)_!"] & \\
\mathcal{F}_U \arrow[ur, "s_1^*"] \arrow[dr, "s_2^*"'] \arrow[rr, phantom, "\Downarrow\tilde f"] & & \mathcal{F}_V \arrow[ur, "(s'_1)^*"'] \arrow[dr, "(s'_2)^*"'] \arrow[rr, phantom, "\Downarrow\tilde g"] & & \mathcal{F}_W \\
 & \mathcal{F}_{T_1} \arrow[dr, "\pi_1^*"'] \arrow[ur, "(t_2)_!"'] & & \mathcal{F}_{T_2} \arrow[ur, "(t'_2)_!"'] & \\
 & & \mathcal{F}_{(T_1\times_V T_2)} \arrow[ur, "(\pi_2)_!"'] & &
\end{tikzcd}$$

```
$$\xymatrix@!=3.4pc{
d^*_0X\approxeq k^*_0(d^*_2d^*_1X)
\ar[rrrr]^{k^*_0(d^*_2\theta)}
\ar[dr]^{c(k_0,d^*_2d^*_1X)}
\ar@/_1.6pc/[dddrr]_1 &&&&
k^*_0(d^*_2d^*_0X)\approxeq d^*_1X
\ar[dl]_{c(k_0,d^*_2d^*_0X)}
\ar@/^1.6pc/[dddll]^\theta\\
& d^*_2d^*_1X\approxeq d^*_1d^*_1X
\ar[rr]^{d^*_2\theta}  \ar[dr]_{d^*_1\theta}& &
d^*_2d^*_0X\approxeq d^*_0d^*_1X
\ar[dl]^{d^*_0\theta} &\\
&& d^*_1d^*_0X\approxeq d^*_0d^*_0X &&\\
&& k^*_0(d^*_0d^*_0X)\approxeq d^*_0X
\ar[u]|{c(k_0,d^*_0d^*_0X)}&&}$$
```

$$\begin{tikzcd}
d^*_0X\approxeq k^*_0(d^*_2d^*_1X) \arrow[rrrr, "k^*_0(d^*_2\theta)"] \arrow[dr, "c(k_0,d^*_2d^*_1X)"] \arrow[dddrr, bend right, "1"'] & & & & k^*_0(d^*_2d^*_0X)\approxeq d^*_1X \arrow[dl, "c(k_0,d^*_2d^*_0X)"'] \arrow[dddll, bend left, "\theta"] \\
& d^*_2d^*_1X\approxeq d^*_1d^*_1X \arrow[rr, "d^*_2\theta"] \arrow[dr, "d^*_1\theta"'] & & d^*_2d^*_0X\approxeq d^*_0d^*_1X \arrow[dl, "d^*_0\theta"] & \\
& & d^*_1d^*_0X\approxeq d^*_0d^*_0X & & \\
& & k^*_0(d^*_0d^*_0X)\approxeq d^*_0X \arrow[u, "c(k_0,d^*_0d^*_0X)" description] & &
\end{tikzcd}$$

```
Let $N=\big\{m\in M\; |\; g(1\otimes m)=m\otimes 1\big\}$, and
observe that $N$ is the kernel of the pair
$(\varepsilon_2,g\varepsilon_1) \; :$
$$\xymatrixcolsep{2pc}
\xymatrix{
0\ar[r] &N\ar[r] &M\ar@<.7ex>[r]^-{\varepsilon_2}
\ar@<-.7ex>[r]_-{g\varepsilon_1}& **[r]M\otimes S\;,}$$
where the pair
$(\varepsilon_2,g\varepsilon_1)$ is as shown:
$$\xymatrix{
   M \ar@/^1pc/[rr]^{\epsilon_2}\ar[r]|-{\varepsilon_1}
\ar@/_1pc/[rr]_{g\epsilon_1}  &S\otimes M \ar[r]|-g &M\otimes S.}$$
```

Let $N = \big\{m \in M \;|\; g(1 \otimes m) = m \otimes 1\big\}$, and observe that $N$ is the kernel of the pair $(\varepsilon_2, g\varepsilon_1)\;:$

$$\begin{tikzcd} 0 \arrow[r] & N \arrow[r] & M \arrow[r, shift left, "\varepsilon_2"] \arrow[r, shift right, "g\varepsilon_1"'] & M \otimes S\;, \end{tikzcd}$$

where the pair $(\varepsilon_2, g\varepsilon_1)$ is as shown:

$$\begin{tikzcd} M \arrow[rr, bend left, "\epsilon_2"] \arrow[r, "\varepsilon_1" description] \arrow[rr, bend right, "g\epsilon_1"'] & S \otimes M \arrow[r, "g" description] & M \otimes S. \end{tikzcd}$$

```
\[
\xymatrix@1{ A \ar@<+2pt> ‘u[r] ‘[r] [r]
               \ar@<-2pt> ‘u[r] ‘[r] [r] & B }
\]
```

$$\begin{tikzcd} A \arrow[r, bend left=90] \arrow[r, bend left=80] & B \end{tikzcd}$$

```
\[
\xymatrix@1{ A \ar@/^/[r] \ar@/^/@<-1ex>[r] & B }
\]
```

$$\begin{tikzcd} A \arrow[r, bend left] \arrow[r, bend left=15] & B \end{tikzcd}$$

```
\[
\xymatrix@C=3pc{
S_1
\ar@/^2pc/ [r]^{d_1}\ar@/^1pc/ [r]^{d_0}&
S_0\ar@(dr,ur)[]|{id}
\ar@/^1pc/ [l]^{s_0}}
\]
```

$$\begin{tikzcd}[column sep=3pc] S_1 \arrow[r, bend left=50, "d_1"] \arrow[r, bend left=20, "d_0"] & S_0 \arrow[loop right, "id" description] \arrow[l, bend left=20, "s_0"] \end{tikzcd}$$

```
\[
S_\diamond :\qquad \xymatrix{
**[l]\hdots\hdots  S_n
\ar@/^4pc/ [r]^{d_n}_\vdots
\ar@/^1pc/[r]^{d_0}  &
**[r]S_{n-1}\ar@/^1pc/[l]^{s_{n-1}} \ar@/^4pc/ [l]^{s_0}_\vdots &
**[l]\hdots\hdots S_2
\ar@/^3pc/ [r]^{d_2}\ar@/^2pc/ [r]^{d_1}
\ar@/^1pc/ [r]^{d_0}&
**[r]S_1
\ar@/^2pc/ [l]^{s_0}
\ar@/^1pc/ [l]^{s_1}
\ar@/^2pc/ [r]^{d_1}\ar@/^1pc/ [r]^{d_0}&
**[r]S_0
\ar@/^1pc/ [l]^{s_0}
}
\]
```

$$S_\diamond : \qquad \begin{tikzcd}[column sep=large] \cdots\cdots S_n \arrow[r, bend left=70, "d_n", "\vdots"'] \arrow[r, bend left, "d_0"] & S_{n-1} \arrow[l, bend left, "s_{n-1}"] \arrow[l, bend left=70, "s_0", "\vdots"'] & \cdots\cdots S_2 \arrow[r, bend left=60, "d_2"] \arrow[r, bend left=40, "d_1"] \arrow[r, bend left=20, "d_0"] & S_1 \arrow[l, bend left=20, "s_1"] \arrow[l, bend left=40, "s_0"] \arrow[r, bend left=40, "d_1"] \arrow[r, bend left=20, "d_0"] & S_0 \arrow[l, bend left=20, "s_0"] \end{tikzcd}$$

```
$$\xymatrixcolsep{4pc}\xymatrixrowsep{3.9pc}
\xymatrix{
& {\mathcal C}_1\times_{{\mathcal C}_0}X
\ar[r]^-{\epsilon_x}
\ar@/_/[dl]_{{\mathcal C}_1\times_{{\mathcal C}_0}f}
\ar@{.>}[ddl]^(.25){\hskip -8truept d^*_1x}
&X
\ar@/_1pc/[dl]_{f} \ar[ddl]^(.25){x}\\
{\mathcal C}_1\times_{{\mathcal C}_0} Y
\ar[r]^(.73){\epsilon_y} \ar[d]_{d^*_1y}
&Y \ar[d]^{y}\\
{\mathcal C}_1 \ar[r]_{d_1} & {\mathcal C}_0}$$
```

$$\begin{tikzcd}[column sep=4pc, row sep=3.9pc]
& \mathcal{C}_1 \times_{\mathcal{C}_0} X \arrow[r, "\epsilon_x"] \arrow[dl, bend right, "\mathcal{C}_1\times_{\mathcal{C}_0} f"'] \arrow[ddl, dotted, "d_1^* x" near start] & X \arrow[dl, bend right, "f"'] \arrow[ddl, "x" near start] \\
\mathcal{C}_1 \times_{\mathcal{C}_0} Y \arrow[r, "\epsilon_y" near end] \arrow[d, "d_1^* y"'] & Y \arrow[d, "y"] & \\
\mathcal{C}_1 \arrow[r, "d_1"'] & \mathcal{C}_0 &
\end{tikzcd}$$

---

```
$$\xymatrix@!=2.9pc{
X \ar@<-2pt> `u[r] `[rr]^1 [rr] \ar@{.>}[r]^-{\eta_x} \ar[d]_x
\ar@{}[dr]|{\txt{I}}&
{\mathcal C}_1\times_{{\mathcal C}_0}X \ar[r]^-{\epsilon_x}\ar[d]_{d^*_1x}
\ar@{}[dr]|{\txt{II}} & X\ar[d]^x\\
{\mathcal C}_0 \ar[r]^e \ar@<+2pt> `d[r] `[rr]_1 [rr] &{\mathcal C}_1 \ar[r]^{d_1}
{\mathcal C}_0}$$
```

$$\begin{tikzcd}[sep=2.9pc]
X \arrow[rr, bend left, "1"] \arrow[r, dotted, "\eta_x"] \arrow[d, "x"'] \arrow[dr, phantom, "\mathrm{I}"] & \mathcal{C}_1 \times_{\mathcal{C}_0} X \arrow[r, "\epsilon_x"] \arrow[d, "d_1^* x"'] \arrow[dr, phantom, "\mathrm{II}"] & X \arrow[d, "x"] \\
\mathcal{C}_0 \arrow[r, "e"] \arrow[rr, bend right, "1"'] & \mathcal{C}_1 \arrow[r, "d_1"] & \mathcal{C}_0
\end{tikzcd}$$

## 2.3 (ROTATION)

Diagrams can be displayed rotated at any direction. The commutative diagram,

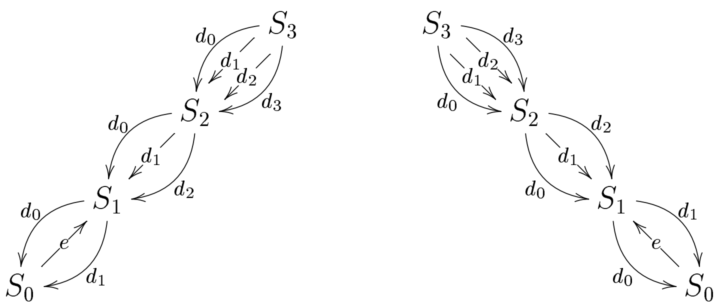
```
$$
\xymatrix{
S_3\ruppertwocell<5>^{d_3}{\omit}\ar@<1ex>[r]|{d_2}\ar@<-1ex>[r]|{d_1}
\rlowertwocell<-5>_{d_0}{\omit}&   S_2 \ruppertwocell<5>^{d_2}{\omit}
\ar[r]|{d_1}\rlowertwocell<-5>_{d_0}{\omit} &S_1
\ruppertwocell<5>^{d_1}{\omit} \rlowertwocell<-5>_{d_0}{\omit} &S_0\ar[l]|e}
$$
```

$$
\begin{tikzcd}
S_3 \arrow[r, bend left, "d_3"] \arrow[r, shift left, "d_2" description] \arrow[r, shift right, "d_1" description] \arrow[r, bend right, "d_0"'] & S_2 \arrow[r, bend left, "d_2"] \arrow[r, "d_1" description] \arrow[r, bend right, "d_0"'] & S_1 \arrow[r, bend left, "d_1"] \arrow[r, bend right, "d_0"'] & S_0 \arrow[l, "e" description]
\end{tikzcd}
$$

for example, can be rotated to the southwest or southeast directions, by simply adding the options `@dl` and `@dr`, respectively, just after the `\xymatrix` command. Thus,

```
$$
\xymatrix@dl{..........} \qquad\qquad \xymatrix@dr{..........}
$$
```

will produce



Here are two more examples from `xyguide.ps`, to study, where the diagrams are rotated as well as scaled.

```
\[
     \xymatrix@dr@C=1pc{
      a \ar[r]\ar@{->>}[d] & a' \ar@{.>>}[d] \\
      b \ar@{.>>}[r]       & b' }
\]
```

$$\begin{tikzcd}[column sep=small, row sep=small] & a \arrow[dl, two heads] \arrow[dr] & \\ b \arrow[dr, dotted, two heads] & & a' \arrow[dl, dotted, two heads] \\ & b' & \end{tikzcd}$$

```
\[
\left(
      \def\objectstyle{\scriptstyle}
      \def\labelstyle{\scriptstyle}
      \vcenter{\xymatrix @-1.2pc @ur {
       A \ar[r]^{a}  & B \ar[d]^{b}  \\
       A'\ar[u]^{a'} & B'\ar[l]^{b'} }}
     \right)
\]
```

$$\left(\begin{tikzcd}[column sep=tiny, row sep=tiny] & B \arrow[dr, "b"] & \\ A \arrow[ur, "a"] & & B' \arrow[dl, "b'"] \\ & A' \arrow[ul, "a'"] & \end{tikzcd}\right)$$