Locating Altered Resistors in Circular and Tetrahedronal Networks

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This is an interim report for the research I completed at the University of Washington. Further research will be completed at Texas A&M University, and a complete, formatted report will follow.

The original purpose of my research was to determine the location of a resistor in a circular network whose conductivity has been altered from its original value. I first attempted to locate the altered resistor by using Robert Coury's method in "Finding an Altered Resistor in a Cubic Network". According to Coury, an altered resistor in a cubic network can be located by determining the angles between the columns of the original lambda matrix and the altered lambda matrix. By determining the maximum angle for each pair of opposing sides, the location of the altered resistor was approximately at the point of intersection of the three rays. However, this process does not completely work for circular networks. First, determining the maximum angle will determine the ray on which the resistor is located (if the altered resistor is located on a ring, there will be two equal maximum angles), but it is not easily determined between which rings the resistor is located. Moreover, there are some unique examples where the altered resistor is not located on the ray with the maximum angle. However, these facts below were determined:

1) By determining the sum of the absolute value of the differences between each column of the lambda matrices, the altered resistor is located on the ray with the maximum sum.
2) By altering an exterior resistor, the angle corresponding to the ray of the resistor equals zero. Thus, the corresponding column in the new lambda matrix is a multiple of the original column.
3) For any (m,3) circular network, altering any radial resistor will result that the corresponding ray will have an angle equal to zero. (i.e. the corresponding column in the new lambda matrix is a multiple of the column of the original lambda matrix.)
4) For any (m,2n+1) circular network (with an odd number of rays), altering a circular resistor results that the angle for the ray directly opposing the altered resistor is zero.

By examining the data of computed angles for various networks, there seemed to be some local patterns for locating radial resistors in (2,n) and (3,n) networks, but these patterns did not hold once n became large.

At this time, I believed that the location of an altered resistor was not recoverable in a circular network, so I began to research tetrahedronal networks. By examining a one-tetrahedronal network, if each conductance of every resistor was arbitrary and one internal resistor was altered, the location of the altered resistor could be determined because it would be located between the rays of the maximum angle and the second maximum angle. However, general tetrahedronal networks with arbitrary conductances do not hold this pattern. In fact, if each tetrahedron has constant conductivity, but different tetrahedrons had different conductivities, I could not locate the position of a uniformly altered tetrahedron. The position of a uniformly altered tetrahedron can only be found if the original network had a constant background conductivity. The location of this altered tetrahedron was located by determining the maximum sum (of differences between the two lambda matrices) for each pair of opposing sides and intersecting the two rays. This same pattern holds for determining the maximum angles if the tetrahedron is not connected to a boundary node.

Further research on multiple-tetrahedronal networks with constant background conductivity resulted that altering two different resistors could result in the same lambda matrix. For every four tetrahedrons connected in a square, altering a diagonal internal resistor conductivity produces the same lambda matrix as altering the diagonal internal resistor of the opposing (diagonal) tetrahedron.

After determining the method for locating a uniformly altered tetrahedron is a network with constant background conductivity, I attempted to apply these methods to circular networks. Since a tetrahedron with four nodes and a cross with five nodes are equivalent networks, I separated each circular network into distinct crosses. With the exception of networks with three rays, I was able to determine the location of every altered cross for networks with one, two, or three rings, for each altered cross produced a unique set of maximum and minimum angles and sums. For four-ringed resistor networks, the uniqueness dissolved, and I intend in the future to determine the cause for this disappearance.

>From examining the patterns resulting by altering a single cross, I was able to return to the data of altering a single resistor and find similar patterns in determining the location of the altered resistor. Because I have not yet discovered all of these relationships, I leave them out of this report. However, I have some new ideas of determining the location of an altered resistor, and hopefully I will get some beneficial results.

My research plans for the future are as follows:
1) Continue my research of a new method on locating an altered resistor in a circular network.
2) Apply Hudelson's methods of finding multiple altered resistors in circular networks.
3) Modify Hudelson's methods to determine the locations of altered resistors by using angles instead of sums.

For the next two weeks, I can be reached at the following e-mail address: jow@tenet.edu.

I shall contact you when I achieve more results.

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