# DIFFERENTIAL GEOMETRY/PDE SEMINAR

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## Inverse spectral problems for elliptic operators

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The subject of this talk concerns to the classical inverse spectral problem. This inverse problem can be formulated as follows: do the Dirichlet eigenvalues and the derivatives (which order?) of the normalized eigenfunctions at the boundary determine uniquely the coefficients of the corresponding differential operator? For operators of order two this type of theorem is called Borg-Levinson theorem. In the case of the Schrödinger operators the knowledge of the Dirichlet eigenvalues and the normal derivatives of the normalized eigenfunctions at the boundary uniquely determine unknown potential.

Borg-Levinson theorem for the Schrödinger operators was proved for the first time by Nachman, Sylvester and Uhlmann for bounded potentials. The same result was obtained independently by Novikov. For singular (meaning not bounded) potentials from the space $L^p$, $\frac{n}{2} < p < \infty$, this theorem was proved by Päivärinta and Serov. For the magnetic Schrödinger operator with singular coefficients Borg-Levinson theorem was proved for the first time by Serov. For Riemannian manifolds this result was proved by Katchalov, Kurylev and Lassas (they call this problem as the Gelfand inverse problem for quadratic pencil). For Dirac operator Salo and Tzou proved that a Lipschitz continuos magnetic field and electric potential can be uniquely recovered from boundary measurements. For zero order perturbation of the bi-harmonic operator Ikehata proved that the Dirichlet-to-Neumann map uniquely determines the potential. For elliptic partial differential operator (with constant coefficients) with $L^\infty$-potentials Borg-Levinson theorem was proved by Krupchyk and Päivärinta. For the operator of order 4 which is the first order perturbation of the bi-harmonic operator with Navier boundary conditions on a smooth bounded domain it is proved by Krupchyk, Lassas and Uhlmann that the Dirichlet-to-Neumann map uniquely determines this first order perturbation.

We prove that the boundary spectral data, i.e. the Dirichlet eigenvalues and special derivatives of the normalized eigenfunctions at the boundary uniquely determine the coefficients of the magnetic Schrödinger operator and the coefficients of some perturbation of the bi-harmonic operator.

It might be mentioned that usually the authors of mentioned works assume the knowledge of the Dirichlet-to-Neumann map. In the comparison with this we prove that the Dirichlet-to-Neumann map is determined uniquely by our spectral data (Borg-Levinson data). This result has an independent interest.

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