DIFFERENTIAL GEOMETRY/PDE SEMINAR

WEDNESDAY, MAY 7, 2014
PADELFORD C-36
3:50PM–5PM

A Survey of Sub-Riemannian Geometry

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I begin with the problem which introduced me to this subject: How can it be that a cat, dropped from upside down with zero angular momentum, can land on his feet? Appropriately understood, the falling cat problem becomes a problem of finding geodesics for a sub-Riemannian metric. Such a metric is a pair (non-integrable distribution, inner product on distribution) where by "distribution" I mean k-plane field: a rank k linear subbundle of the tangent bundle of a manifold. These metrics arise naturally as limits of Riemannian manifolds, in the theory of groups of polynomial growth, in optimal control, and in the study of sub-elliptic operators. My plan is to build up from the simplest example - the three-dimensional Heisenberg group - into some of the more complicated phenomena arising in these geometries: singular geodesics, moduli of inequivalent local models, impossiblity of Lipshitz embeddings of the Heisenberg group into Hilbert space, the Sard theorem for the endpoint map, and the problems of understanding "sub-Riemannian curvature". With some luck I may be able to give some insights on theorems and phenomena established since I wrote my book on the subject.