Assignment 2. Due Fri., Oct. 10.

Reading: Sec. 1.62 and 1.64.

1. p. 66, problems 9 and 21; p. 70, problem 1 under Exercises; p. 70, problem 5 under Miscellaneous Exercises.

2. A function g defined on $\mathbf{R}$ is a *contraction* if there is a number $L < 1$ such that for any points x and y in $\mathbf{R}$,
$$|g(x) - g(y)| \leq L \cdot |x - y|.$$
   (a) Show that such a function is *continuous* at all points $x_0 \in \mathbf{R}$.

   (b) It can also be shown that a contraction g has a unique *fixed point*; that is, there is one and only one number A such that $g(A) = A$. [You may use this fact; you do not need to show it.]

   Let x_1 be any real number and consider the sequence $\{x_n\}$ defined by $x_{n+1} = g(x_n)$, $n = 1, 2, \ldots$. Show that this sequence has a limit and that the limit is the fixed point A of g. [Hint: Show that the sequence $|x_{n+1} - A| = |g(x_n) - g(A)|$ converges to 0 as $n \to \infty$.]