Assignment 1. Due Friday, Apr. 10.

Reading: Section on "Applications" in 555 Notes.
p. 1-10 in 556 Notes.

1. Consider an insulated rod so that no heat flows out the ends of the rod.

   (a) Convince yourself that this gives rise to the problem: DE: $u_t = u_{xx}$, $0 \le x \le \pi$, $t \ge 0$; IC: $u(x,0) = f(x)$, $0 \le x \le \pi$; BC: $u_x(0,t) = u_x(\pi,t) = 0$, $t \ge 0$.

   (b) Separate variables to find the fundamental modes $u(x,t)$.

   (c) Show that the resulting initial states form a complete orthonormal system in $L^2(0,\pi)$. (The associated series are Fourier cosine series.)

2. Consider the Dirichlet problem for the Laplacian $\triangle = \partial_x^2 + \partial_y^2$ on the unit disk $D = \{(x,y) : x^2 + y^2 \le 1\}$; i.e., given f on $\partial D = S^1$, find u on D satisfying $\triangle u = 0$, $u\,|_{\partial D} = f$.

   (a) Write $\triangle$ in polar coordinates (r,θ) and separate variables to find solutions of the form $u(r,\theta) = v(r)w(\theta)$. (Note: u should be bounded near the origin. To solve the equation for v, look up Euler equations in an ODE book.)

   (b) Suppose $f \in L^2(S^1)$. Write f in a Fourier series and derive a series for u. Show that this series converges for $r < 1$ to a C^2 solution of $\triangle u = 0$ which satisfies the BC in the sense that $\|u(r,\cdot) - f(\cdot)\|_{L^2(S^1)} \to 0$ as $r \to 1$.

3. Do problems 1 and 2 on Problem Set 1 in the 556 Notes.