Assignment 1.

Due Mon., Jan. 22.

Objectives: To program a 1-D finite element method and see how it compares with a finite difference approach.

(1) Derive the equations for the continuous piecewise linear finite element approximation to the solution of the boundary value problem

$$Lu \equiv -\frac{d}{dx}\left(p(x)\frac{du}{dx}\right) + q(x)u = f(x), \quad 0 \le x \le \pi,$$

$$u(0) = u(\pi) = 0,$$

starting from the *weak form* of the equation: Find $u \in H_0^1$ such that

$$\langle Lu, v\rangle = \langle f, v\rangle \quad \forall v \in H_0^1,$$

where

$$H_0^1 = \{v : \ v(0) = v(\pi) = 0 \text{ and } \int_0^\pi (v')^2 + v^2 < \infty\}, \quad \langle v, w\rangle = \int_0^\pi v(x)w(x)\,dx.$$

(2) Write a code in MATLAB or another language to solve the problem in (1) with $q(x) = 1$ and

$$p(x) = \begin{cases} 1, & 0 \le x \le \pi/3 \\ 100, & \pi/3 < x \le \pi \end{cases}.$$

You can check your code by fixing a solution u and determining what f is. For example, try setting

$$f(x) = \begin{cases} \frac{5}{4}\sin\frac{x}{2}, & x \le \pi/3 \\ \frac{13}{50}\sin\frac{x}{2} + \frac{99}{200}, & x > \pi/3 \end{cases}.$$

Then the solution is

$$u(x) = \begin{cases} \sin\frac{x}{2}, & x \le \pi/3 \\ \frac{1}{100}\sin\frac{x}{2} + \frac{99}{200}, & x > \pi/3 \end{cases}.$$

As you refine the mesh, your approximate solution should approach the true solution. Based on your numerical experiments, what would you say is the *order of accuracy* of this finite element method?

(3) Formulate and implement a finite difference method for the problem in (2) and compare the equations and the accuracy with that of the finite element code.