Assignment 2.

Due Mon., Feb. 5.

Objectives: To program a 2-D finite element method; to practice proving V-ellipticity and its consequences.

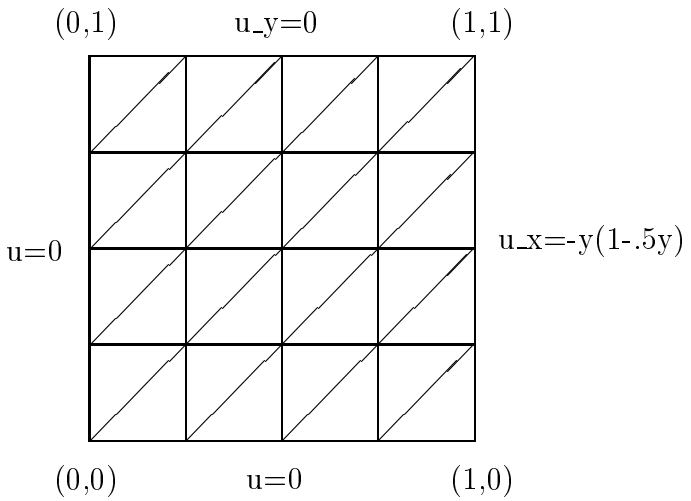
(1) Do problems 2.1, 2.2, 2.6, and 2.7 on pp. 63–64 in the text.

(2) Consider the two-dimensional boundary value problem:

$$-(u_{xx} + u_{yy}) = x(1-x) + y(2-y) \quad \text{in} \quad [0,1] \times [0,1]$$

$$u(x,0) = 0, \quad u_y(x,1) = 0, \quad u(0,y) = 0, \quad u_x(1,y) = -y(1 - \frac{1}{2}y).$$

(If I've done the algebra correctly, the solution is $u(x,y) = x(1-x)y(1-.5y)$.) Write a program to compute the continuous piecewise linear finite element approximation, when the region is divided into regular triangles as pictured below:



Write down the variational formulation of this problem and show that it is V-elliptic. Does the H^1-norm of the error in your computed solution seem to be of order h?