Worksheet 8 Math 126

In Chapter 12 we found a formula for the distance between two skew lines, but we didn't actually find the points on the lines that were the closest together. For this worksheet, we will apply the techniques of Section 14.7 to find the closest points on two skew lines.

1. Let L_1 be the line with Cartesian equations $x = y = z$ and L_2 the line with Cartesian equations $x = 0$, $2z + 2 = y$. Find parametric equations for the two lines, using s as the parameter for L_1 and t as the parameter for L_2, and prove that they are skew.

2. Write down a formula for the distance $d(s, t)$ between the point corresponding to s in L_1 and the point corresponding to t in L_2.

3. We want to minimize the function d, but as usual we would like to avoid dealing the annoying square root in the formula. So we define $f(s, t) = (d(s, t))^2$, and use the fact that d will be a minimum exactly when f is a minimum. The minimum value of d is different from the minimum value of f, but they both occur at the same point (s, t). Find the critical point(s) of $f(s, t)$.

4. Why does the global minimum for f occur at a critical point? (Warning: Problem 36 in §14.7 gives an example with only one critical point, the critical point is a local minimum, but it is not a global minimum.)

5. Compute all of the second partial derivatives at the critical point(s), and apply the second derivatives test on p. 954 to classify the critical point(s). (If you don't remember the test and don't have your book handy, ask your TA to write the second derivatives test on the board.) Notice that, according to step 4, there should be at least one local minimum for f.

6. Answer the original question: Which points on the two lines are closest together? If you want to check your answer, you can compute the distance between the two lines using the methods from Chapter 12.

7. Challenge: The closest points on the skew lines can be found by vector methods, without taking any derivatives. Try this if you finish the steps above before the end of class, or at home if you would like an interesting vector review problem.