# UW Math Circle

October 31, 2023

A *polyhedron* is a three-dimensional shape whose sides are flat polygons. The polygon sides of a polyhedron are called *faces*, the lines where two faces meet are called *edges*, and the points where three or more faces meet are called *vertices*.

1. Punch out your five paper shapes and fold them into polyhedra. Then complete the table below, where $F$ is the number of faces in a polyhedron, $E$ is the number of edges, and $V$ is the number of vertices.

| **F** | **E** | **V** | **F + V** |
|---|---|---|---|
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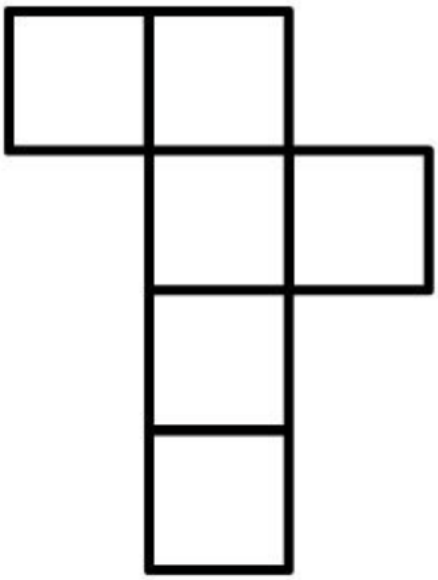
   Can you see a pattern in the numbers you found? Do you think the pattern holds for all polyhedra? Why or why not?

2. A *net* is a flat shape that you can fold into a polyhedron. You saw a net for a cube in Problem 1. Here's another net for a cube:



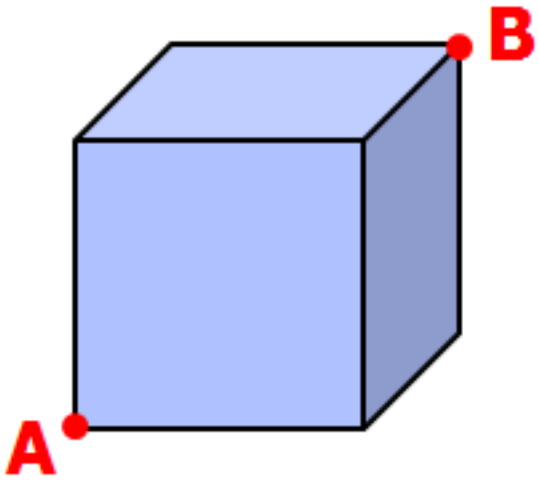
Ignoring rotations and reflections, how many different nets for a cube can you find?

3. Can you construct a polyhedron with five faces? Seven faces? Which positive integers can be the number of faces in some polyhedron?

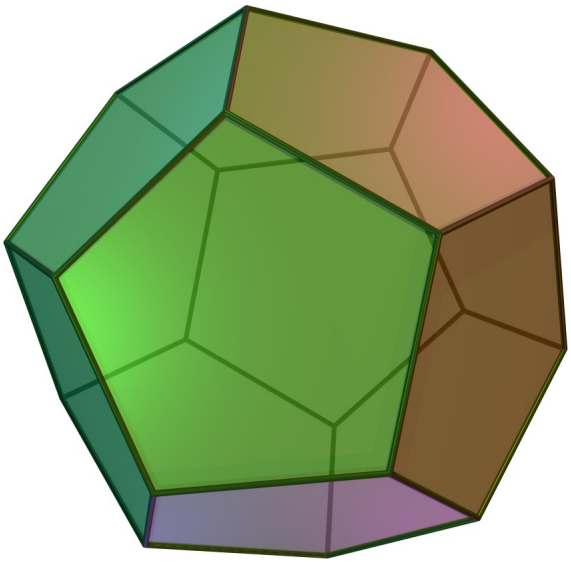
4. Dr. Frankenstein constructs a polyhedron with 62 faces. 20 of its faces are triangles, 30 are squares, and 12 are pentagons. How many edges and vertices does the polyhedron have? (Hint: you don't actually need to construct the polyhedron - use the pattern from Problem 1!)

5. Is it possible to construct a polyhedron from only regular heptagons? Why or why not? What if the heptagons don't have to be regular?

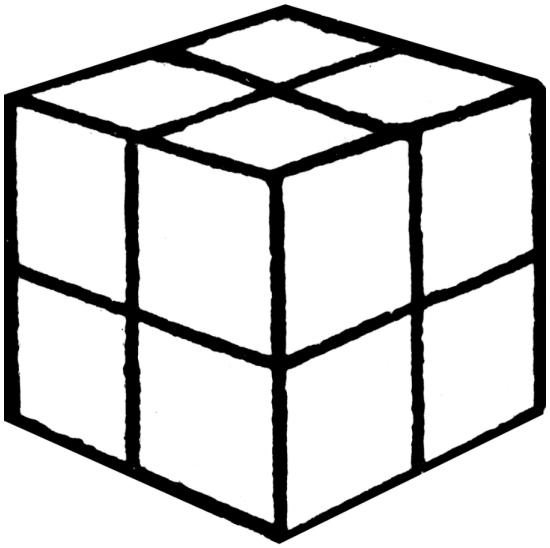
6. Sam the spider sits on vertex $A$ of the cube below. He plans to visit his friend Sally, who lives on vertex $B$. Sam can crawl anywhere on the surface of the cube. What's the shortest distance Sam must crawl to reach Sally? Assume the cube has side length 1 meter.



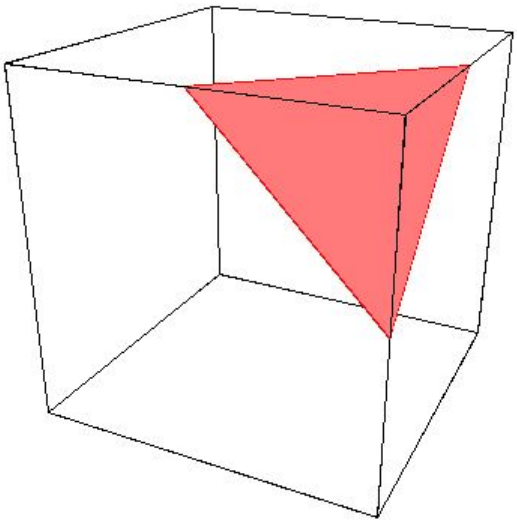
7. Jack Skellington and Oogie Boogie play a game on a dodecahedron. On each player's turn, he places a sticker on an unoccupied vertex of his choice. A player loses if he places a fifth sticker on any face of the dodecahedron. Jack Skellington plays first. Which player has a guaranteed winning strategy?



8. Count Dracula wants to construct a polyhedron out of colored polygons, so that no two polygons of the same color share an edge. How many different colors must Dracula use to construct each polyhedron in Problem 1? How about the polyhedron below, a cube with each face subdivided into four squares?



9. Edward Scissorhands bakes a cake in the shape of a cube. He slices the cake, resulting in a cut shaped like an equilateral triangle.



If Edward had made a different cut instead, which of the following shapes could he have obtained?

- isosceles triangle
- square
- non-square rectangle
- non-square rhombus
- pentagon
- hexagon
- heptagon

Are any other shapes possible?

10. Is it possible to cut a hole through a cube allowing a larger cube to pass through it?