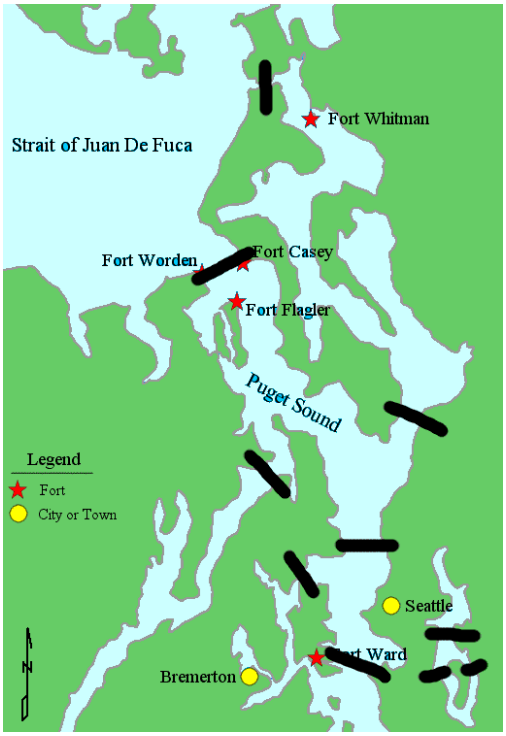
UW Math Circle

Week 11

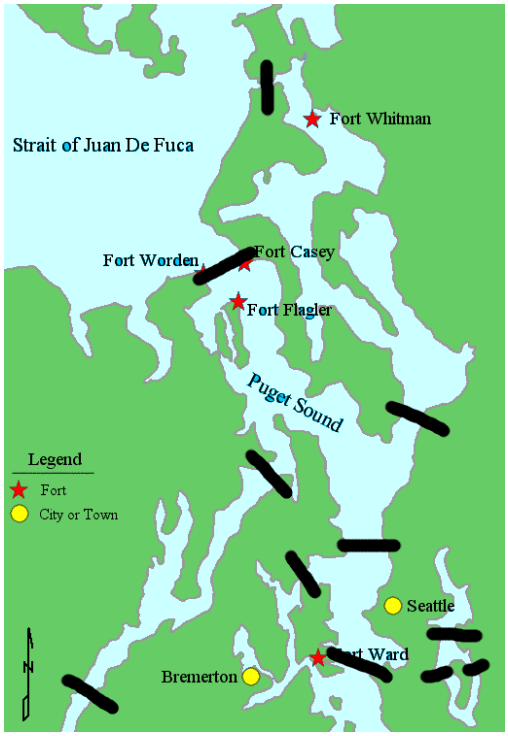
1. What's the maximum possible number of edges in a graph with six vertices? n vertices?

2. A graph is *connected* if you can get from any vertex to any other by traveling along the edges. What's the minimum possible number of edges in a connected graph with six vertices? n vertices?

3. Can you draw a graph with seven vertices, where each vertex belongs to exactly three edges?



(a) The Puget Sound

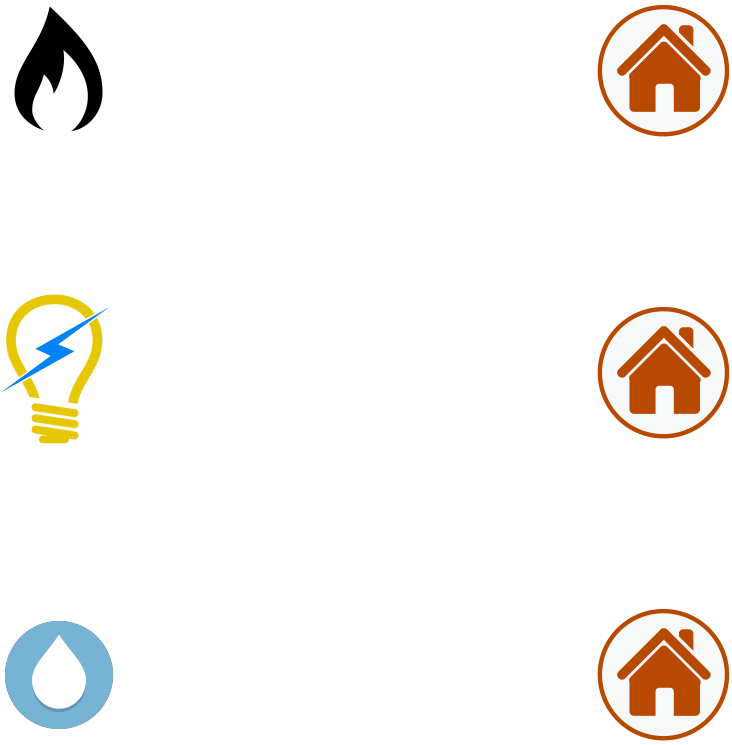


(b) Puget Sound with new bridge

4. (a) The leftmost image above is a map of the Puget Sound with bridges and ferry routes drawn as black lines. Is it possible to traverse a path that uses every bridge and ferry route exactly once? You may start and finish wherever you like, but your path must stay within the map at all times.

   (b) Suppose Kitsap County builds a new bridge in the southwest corner of the map as shown in the rightmost picture below. Now is it possible to traverse a path that uses every bridge and ferry route exactly once?

5. (a) There are three houses and each house must be hooked up to three utilities – water, gas, and electricity. Is it possible to draw supply lines connecting every house to every utility so that none of the supply lines cross?



(b) Now suppose the houses and utilities are on the surface of a sphere. In this case is it possible to draw supply lines connecting every house to every utility so that there are no crossings?

(c) What if each of the houses and utilities lay on the surface of a donut?

6. (Mathcamp 2011 Qualifying Quiz) A country has 7 airports served by 5 airlines. Not every pair of airports is connected by a direct flight, but if you don't mind stopovers, you can get from any airport to any other. Also, if a direct flight between a pair of cities exists, you can travel on it in either direction.

   The airlines are required to coordinate their routes so that if any two of the airlines go bankrupt, a traveler can still get from any airport to any other. What is the smallest number of direct flights that could be offered by all the airlines together?

7. (a) The graph below has vertices numbered 1 through 6. We will travel from vertex 1 to vertex 6 along edges of the graph. Our path doesn't need to visit every vertex, but the vertices it visits must be in ascending order. How many different paths could we take?

   (b) A different graph has vertices numbered 1 to n, where n is an even number. The graph is connected, and each vertex belongs to exactly three edges. Furthermore, if we were to delete any two vertices, the remaining graph would still be connected.

   What is the maximum possible number of ascending-order paths from vertex 1 to vertex n?