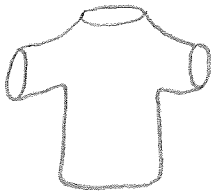
UW Math Circle

Week 14

1. Match the pairs of objects below that are topologically equivalent. For each pair, how would you transform either object into the other?

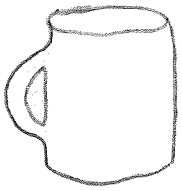
sock



tee-shirt

pants

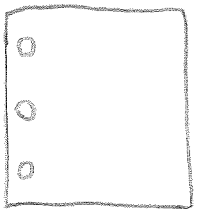
doughnut



coffee cup

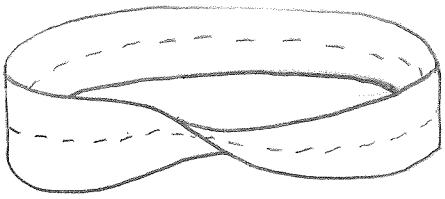
croissant

glasses frame



three-hole-punched piece of paper

2. A Möbius strip is a loop of paper with a half-twist. If you cut the Möbius strip below along the dotted line, what do you predict the resulting pieces will look like?

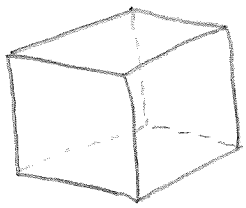


Now, cut a strip of paper and tape it into a Möbius strip. Cut your Möbius strip along the middle. Was your prediction correct?

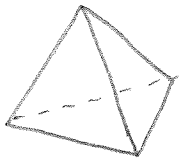
3. a) In a certain county, any two cities are connected directly by a road, and the roads don't cross each other. What is the maximum number of cities this county can have?

b) What if this county is on a Möbius strip? What about a torus?

4. Earlier this year, we learned about Euler's formula: if a polyhedron has V vertices, E edges, and F faces, then $V - E + F = 2$.

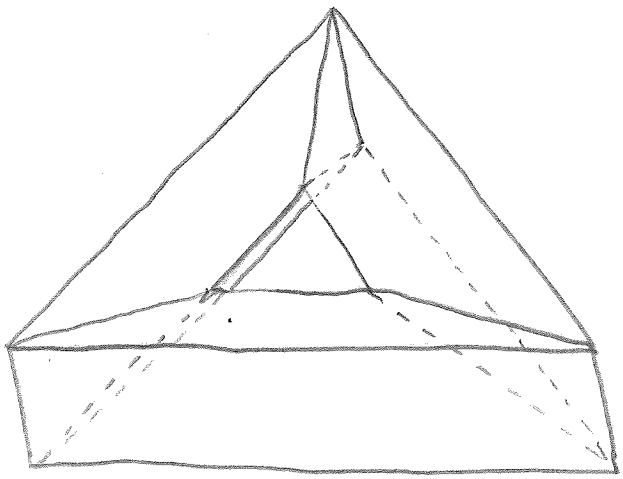


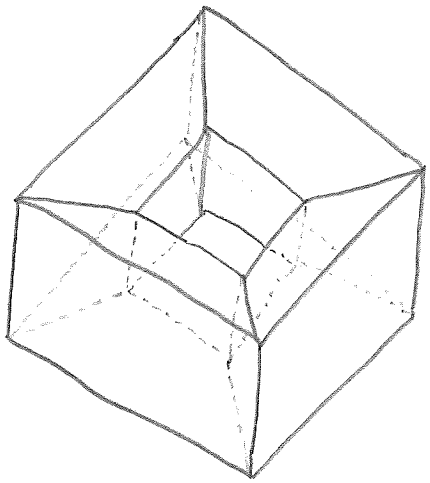
$V = 8$
$E = 12$
$F = 6$



$V = 4$
$E = 6$
$F = 4$

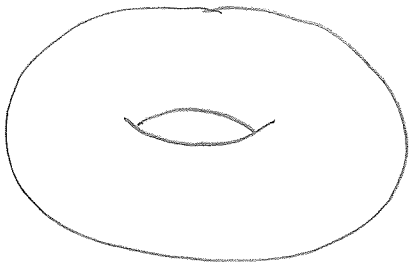
Check if Euler's formula works for the polyhedra below.



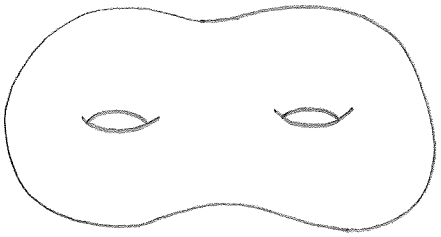


What's different about these polyhedra from the ones that obey Euler's formula? How could you modify Euler's formula so it applies to these polyhedra as well?

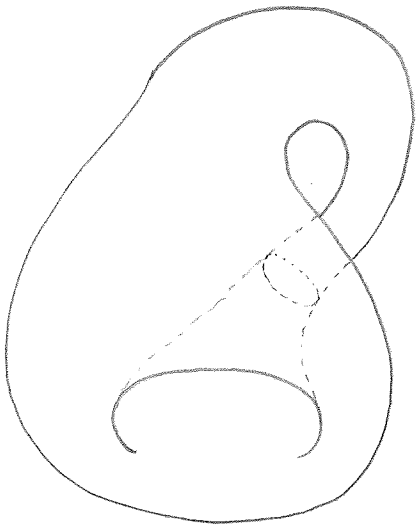
5. Can you draw a fundamental polygon to represent the following shapes?



torus



two-hole torus



Klein bottle

6. The two fundamental polygons below each represent a surface that's pretty hard to draw (though you're welcome to try!)

Are the two surfaces topologically equivalent? Why or why not?