University of Washington Math Hour Olympiad, 2014

Grades 6-7

 Three snails – Alice, Bobby, and Cindy – were racing down a road. Whenever one snail passed another, it waved at the snail it passed.

During the race, Alice waved 3 times and was waved at twice. Bobby waved 4 times and was waved at 3 times. Cindy waved 5 times. How many times was she waved at?



2. Sherlock and Mycroft are playing Battleship on a 4×4 grid. Mycroft hides a single 3×1 cruiser somewhere on the board. Sherlock can pick squares on the grid and fire upon them. What is the smallest number of shots Sherlock has to fire to guarantee at least one hit on the cruiser?



3. Thirty girls – 13 of them in red dresses and 17 in blue dresses – were dancing in a circle, hand-in-hand. Afterwards, each girl was asked if the girl to her right was in a blue dress. Only the girls who had both neighbors in red dresses or both in blue dresses told the truth. How many girls could have answered "Yes"?

4. Herman and Alex play a game on a 5×5 board. On his turn, a player can claim any open square as his territory. Once all the squares are claimed, the winner is the player whose territory has the longer border. Herman goes first. If both play their best, who will win, or will the game end in a draw?



5. Is it possible to find 2014 distinct positive integers whose sum is divisible by each of them?