

# Finding patterns in differences

The mathematics behind the game SET

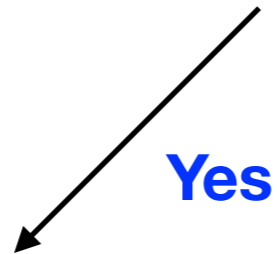
**What is SET?**

# What is SET?

**Is there a card that doesn't belong?**

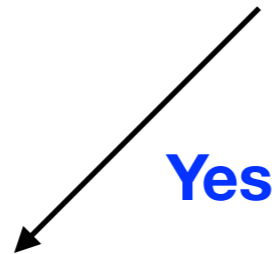
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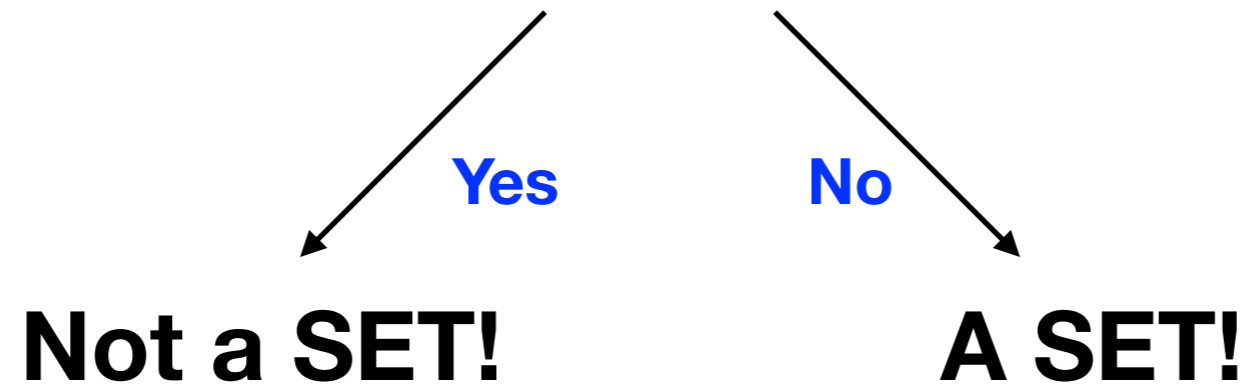
**Is there a card that doesn't belong?**



**Not a SET!**

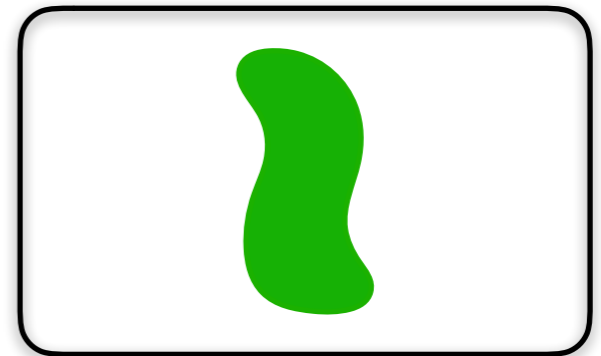
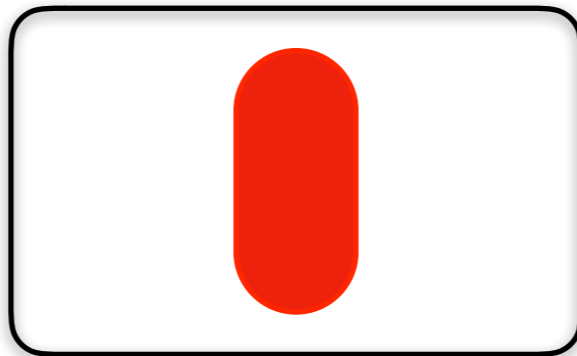
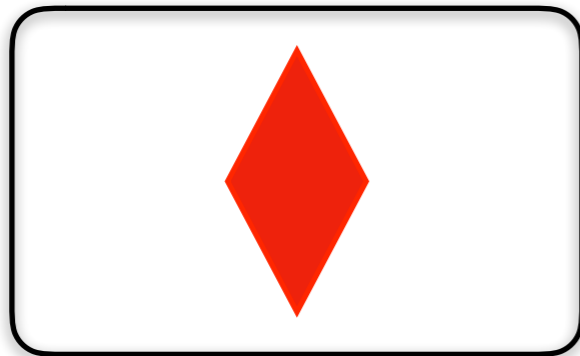
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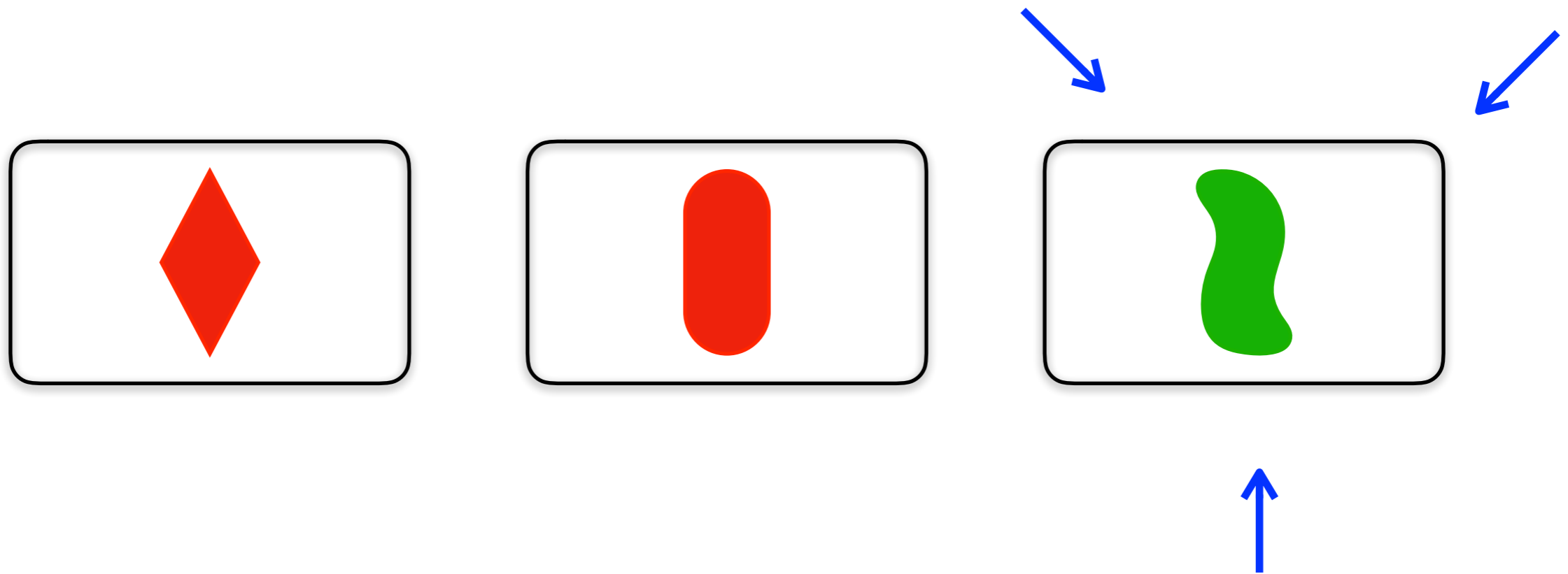
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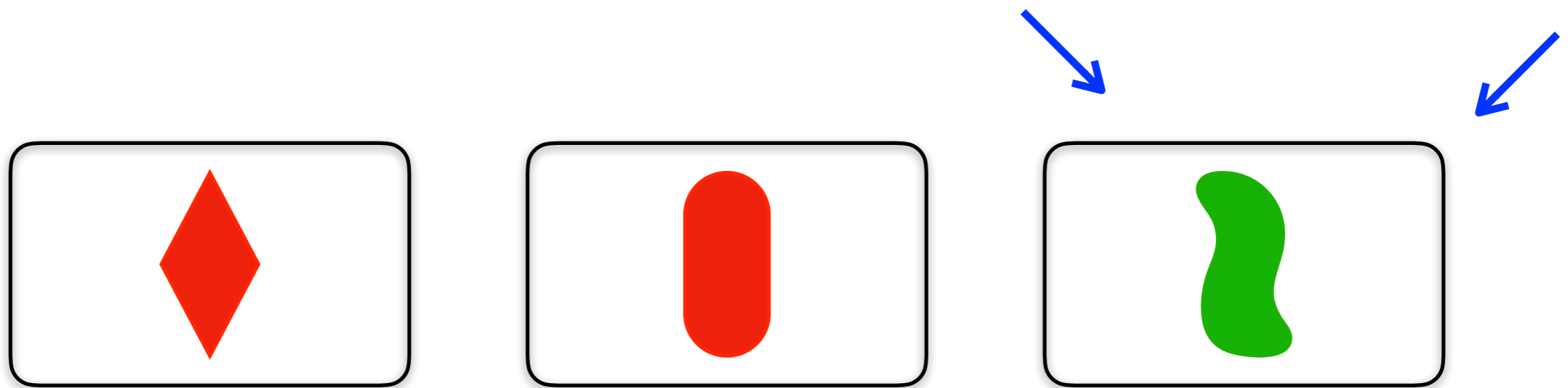
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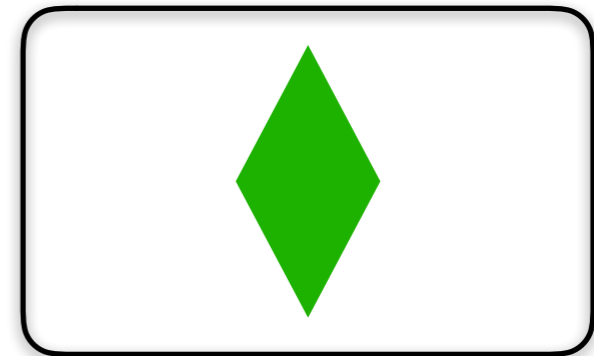
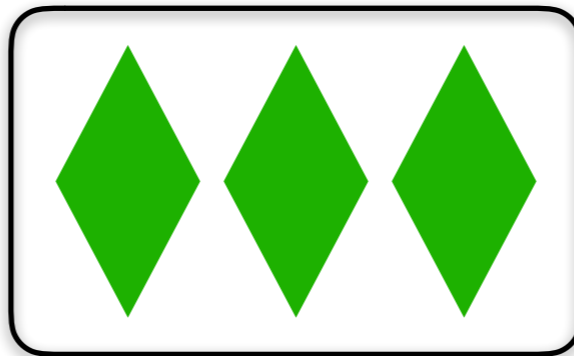
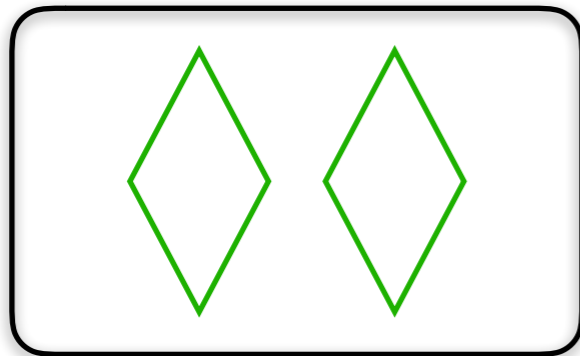
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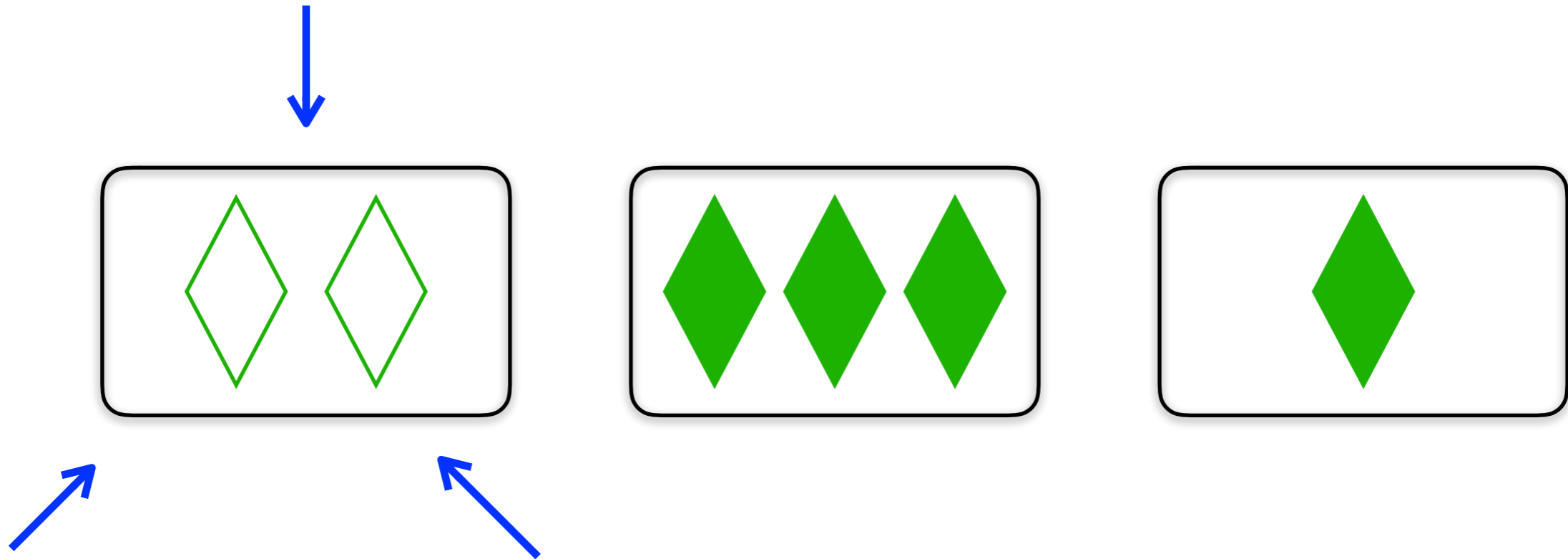
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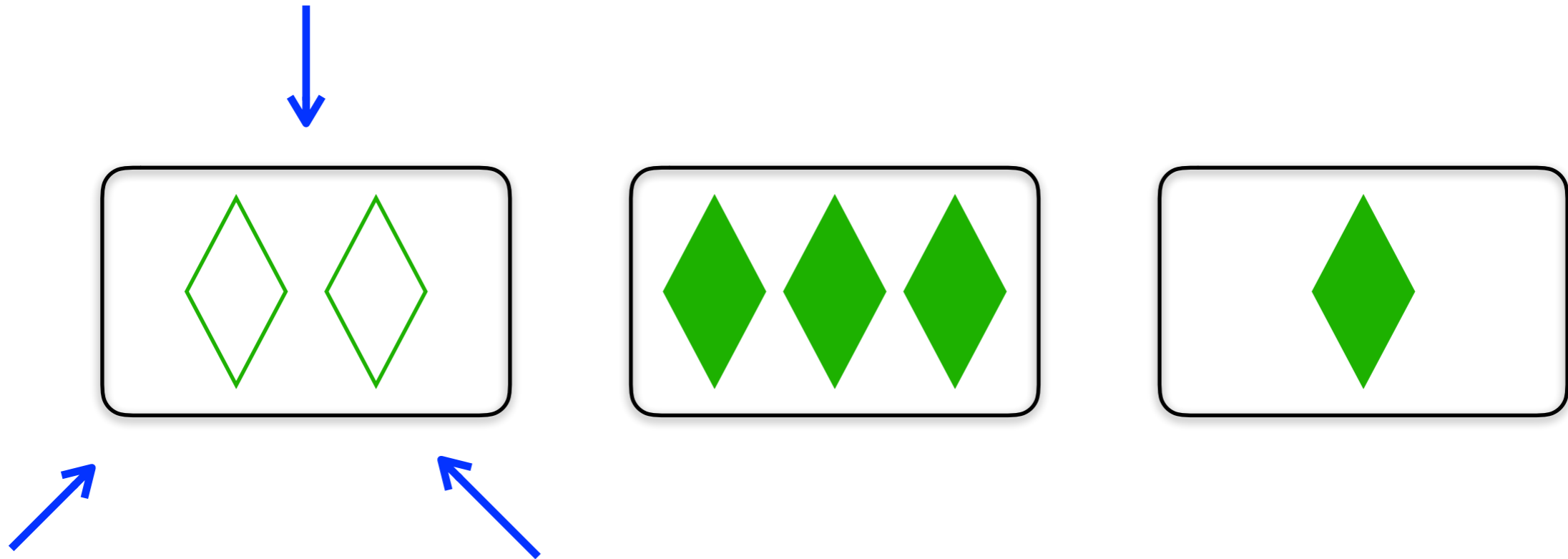
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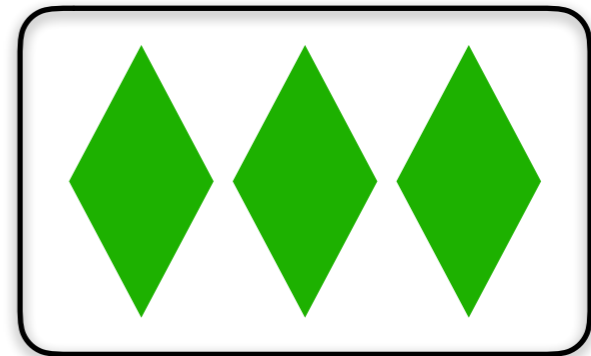
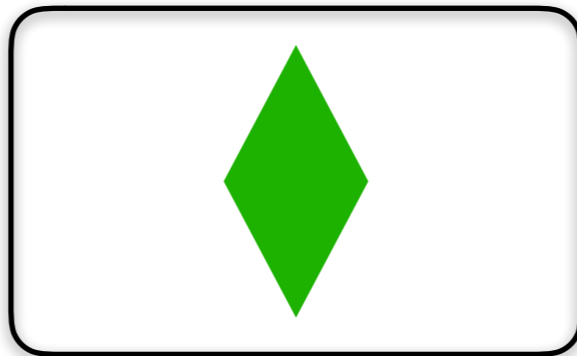
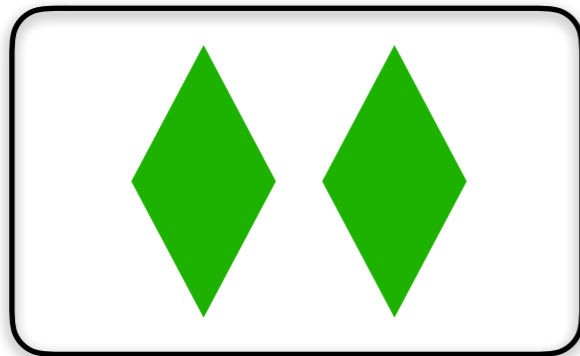
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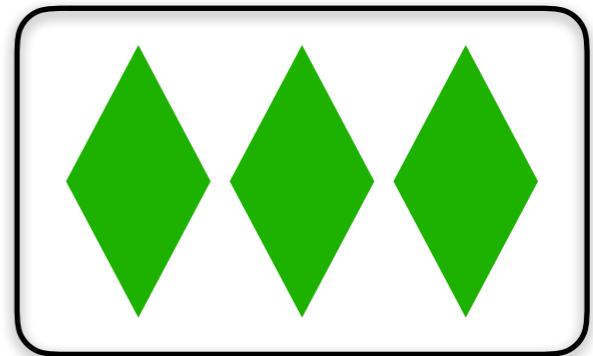
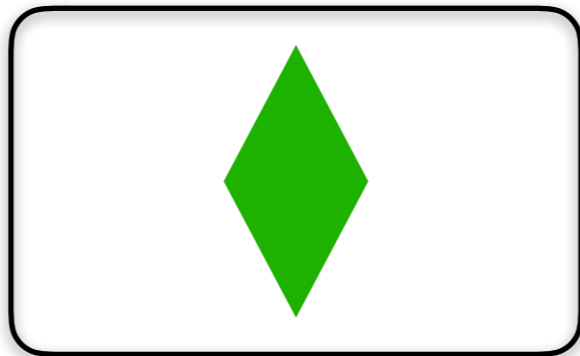
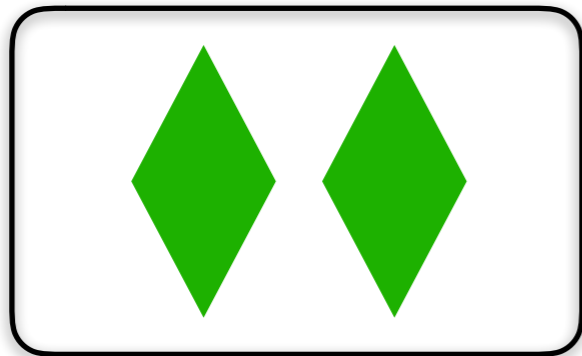
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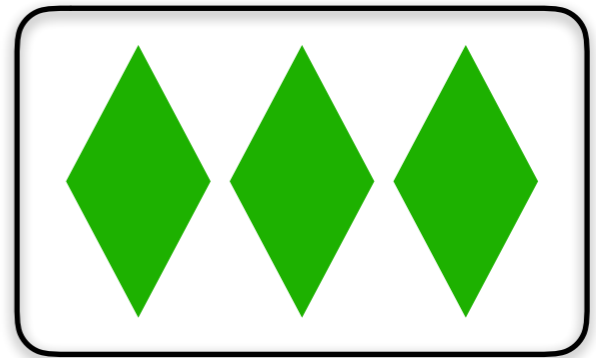
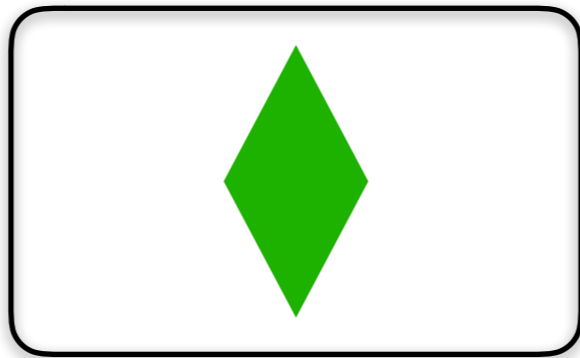
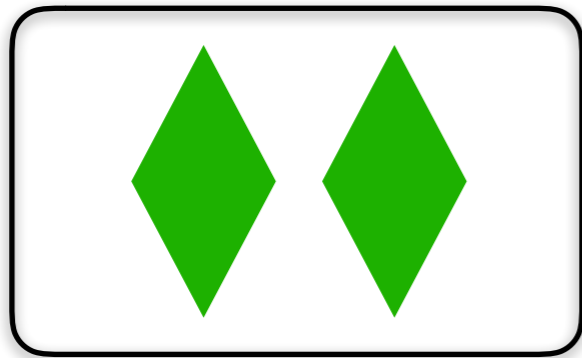
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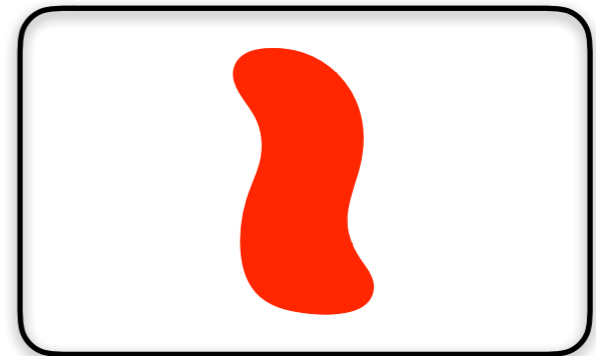
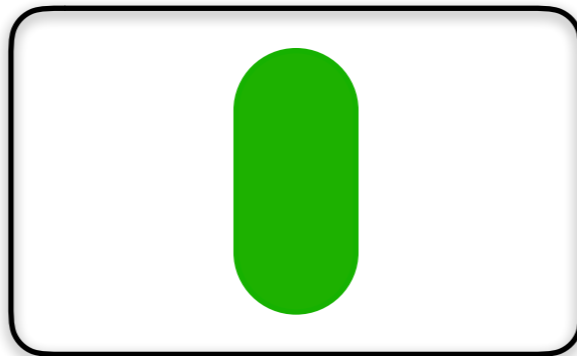
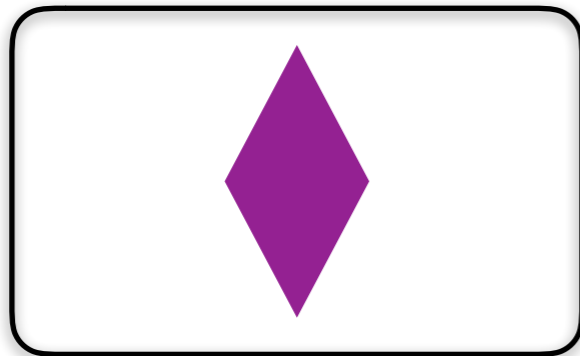
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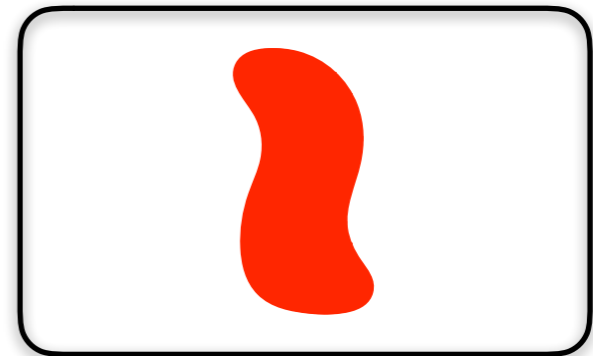
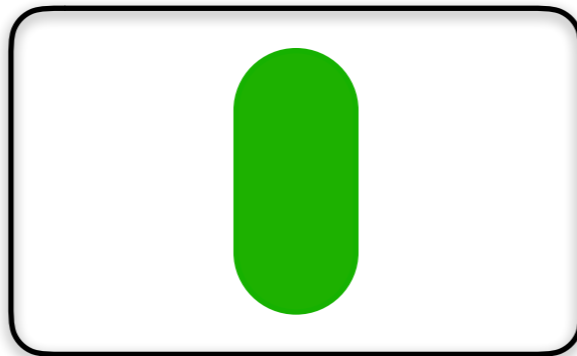
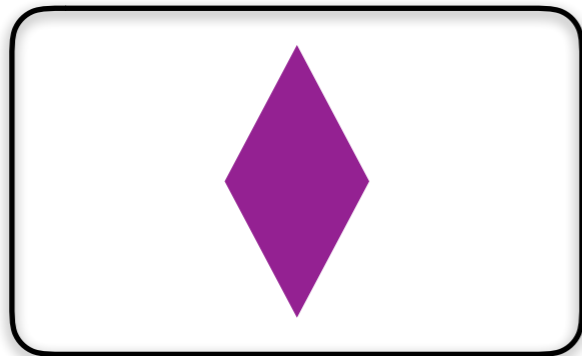
**Is there a card that doesn't belong?**





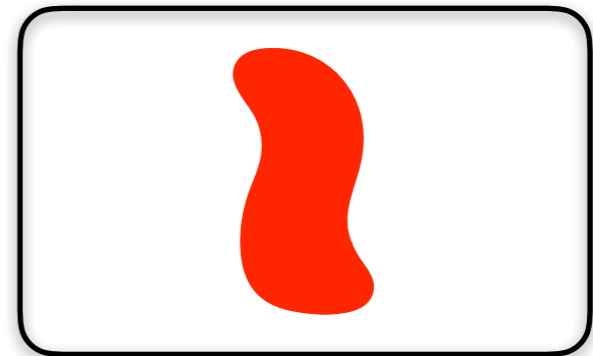
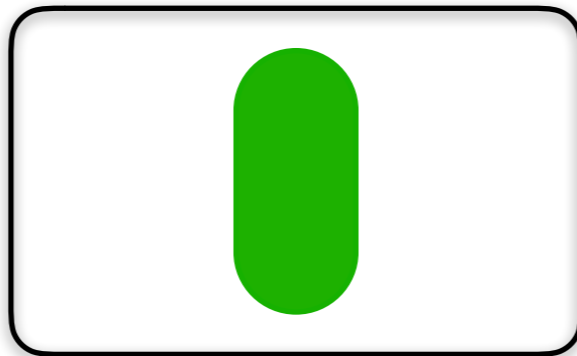
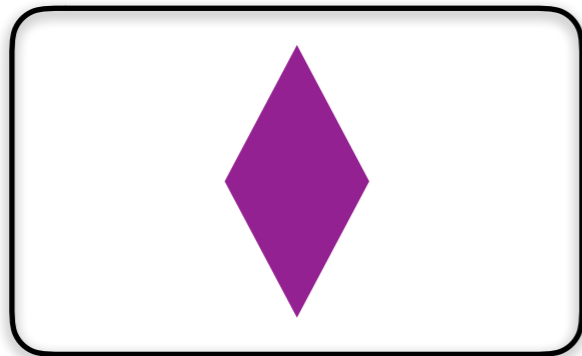
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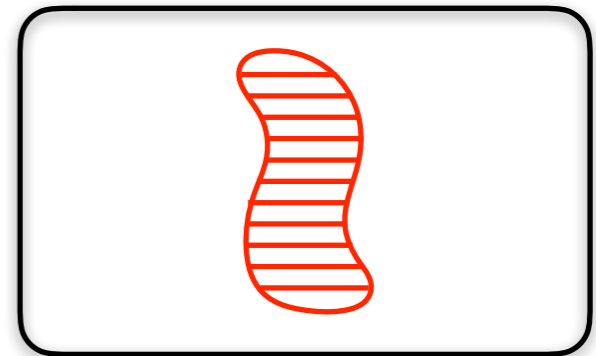
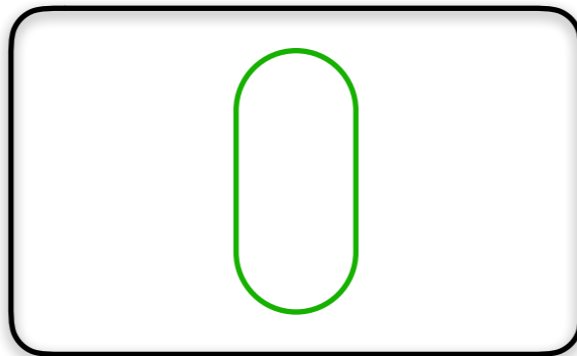
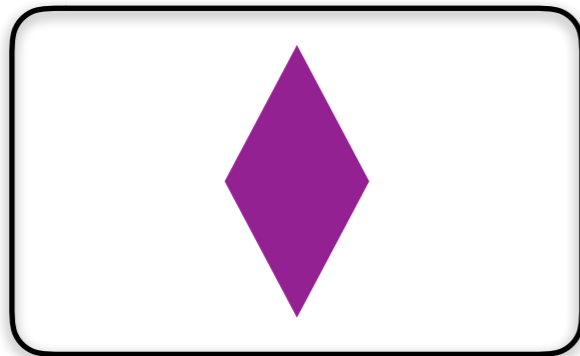
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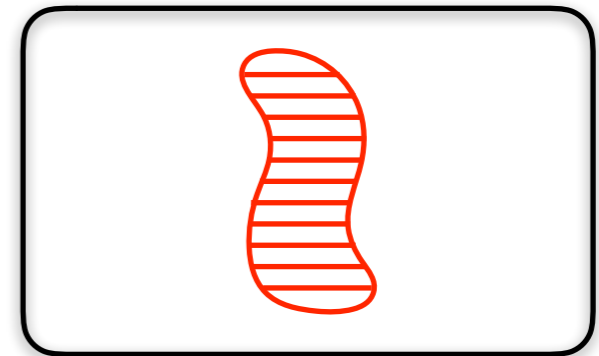
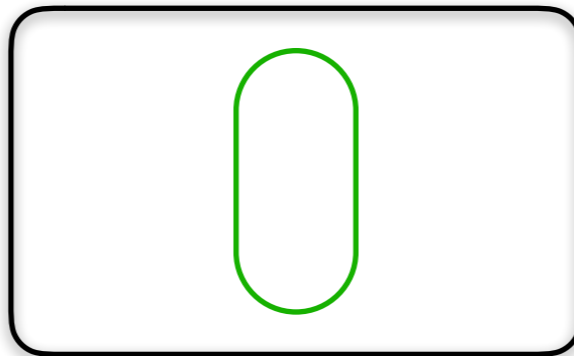
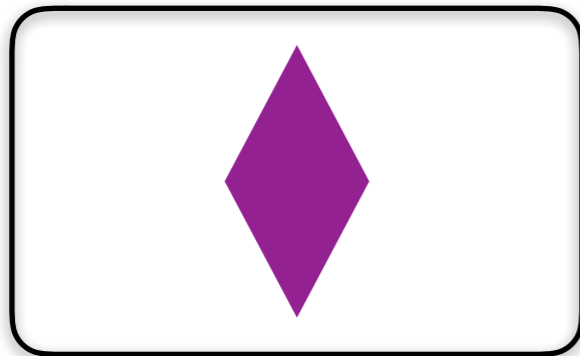
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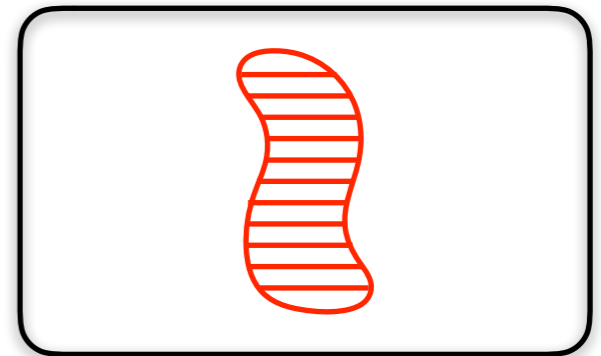
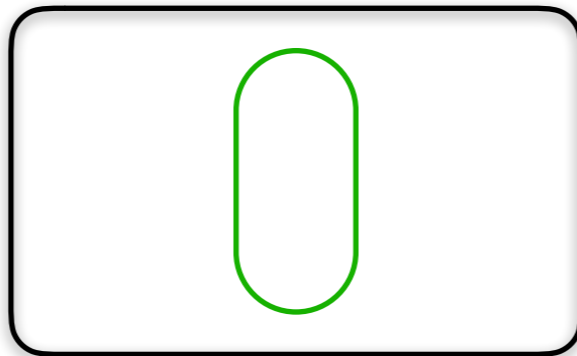
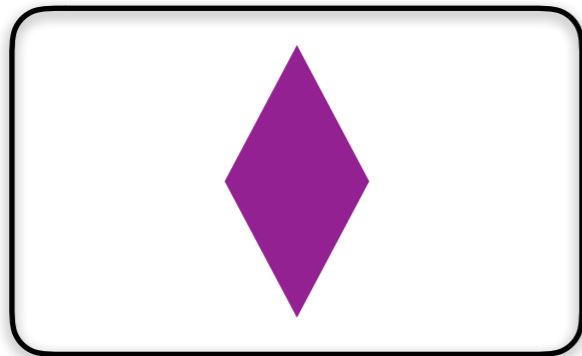
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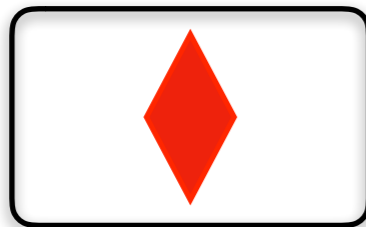
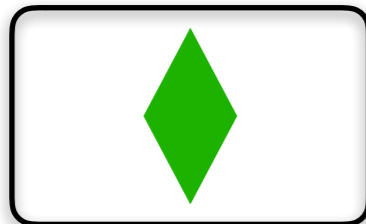
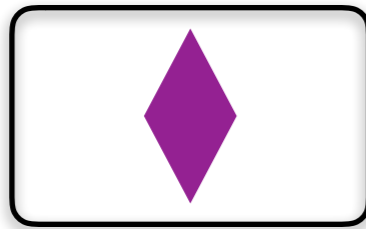
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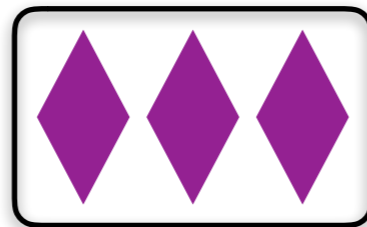
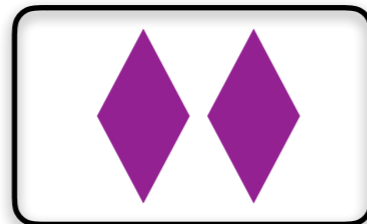
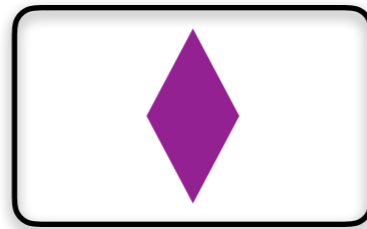
**A SET!!**

# Each card in SET has 4 properties

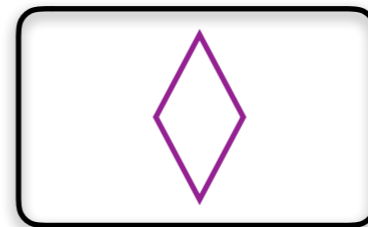
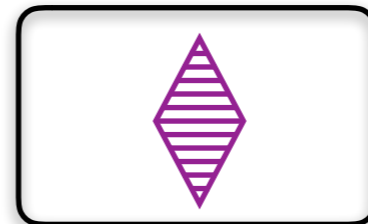
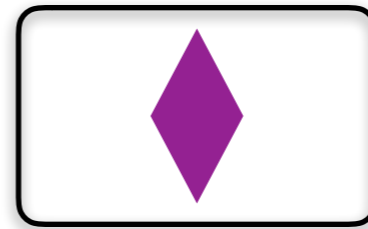
**Color**



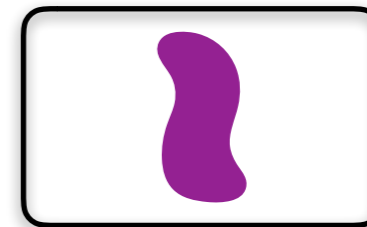
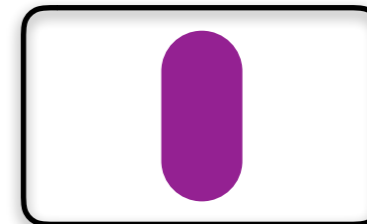
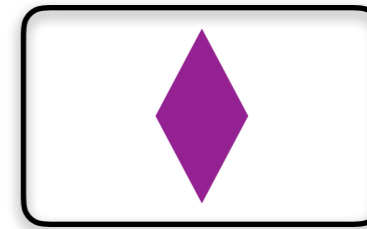
**Number**



**Shading**

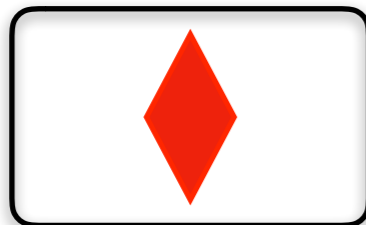
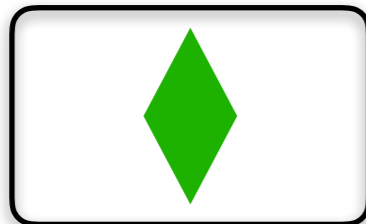
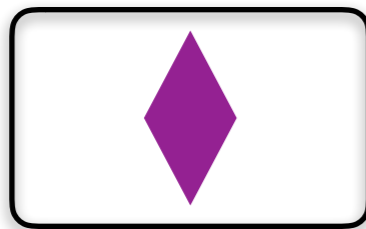


**Shape**

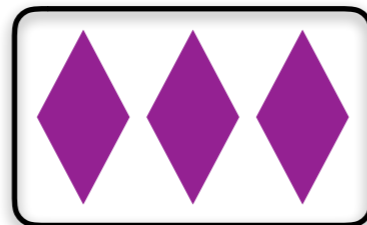
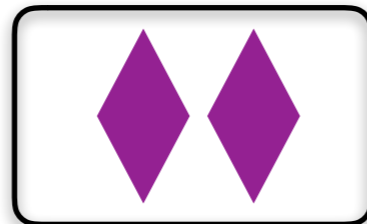
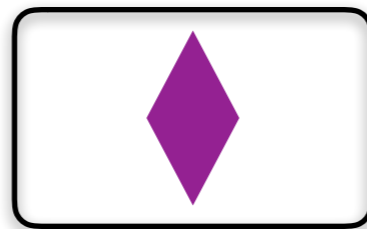


# Each card in SET has 4 properties

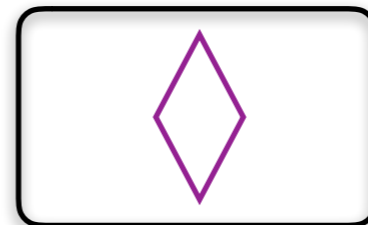
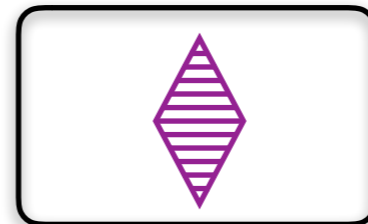
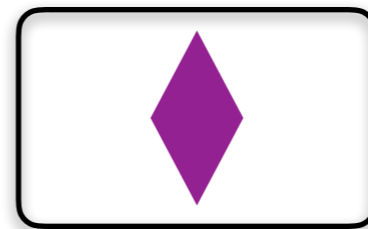
**Color**



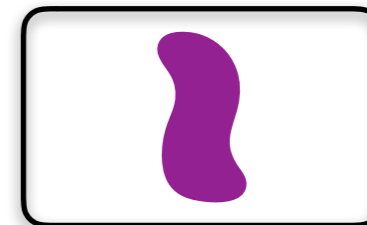
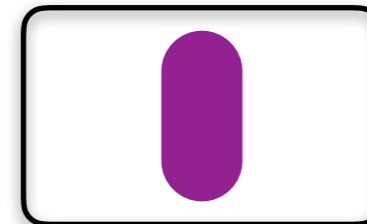
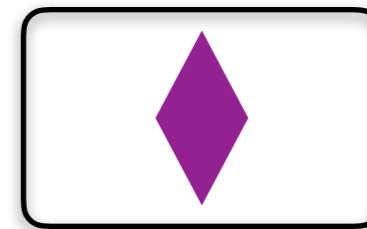
**Number**



**Shading**



**Shape**



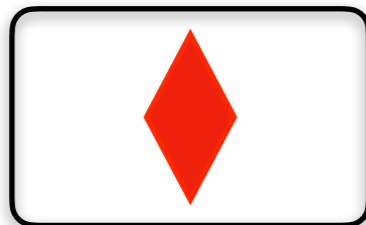
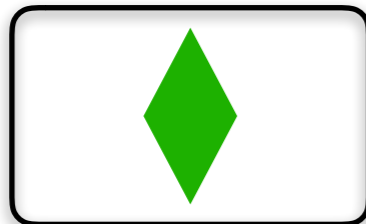
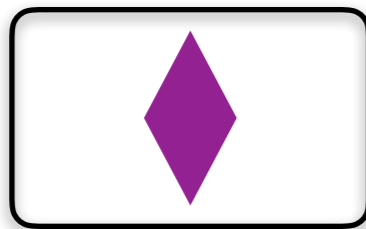
Have  $3 \times 3 \times 3 \times 3 = 3^4 = 81$  cards  
No repeats!

"No card that doesn't belong"

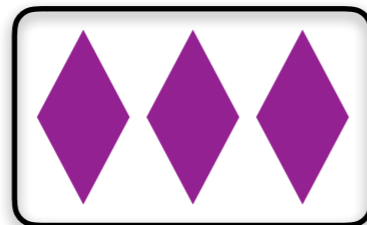
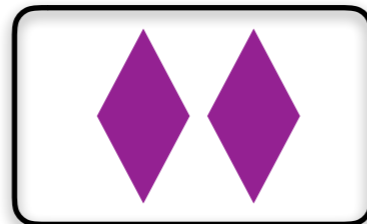
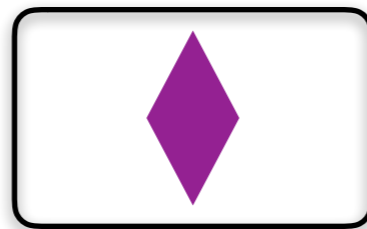


"For every property, all are the same or all are different"

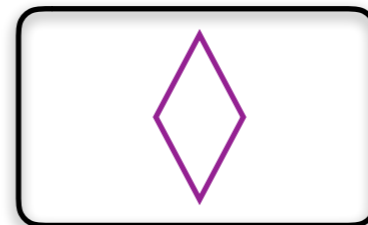
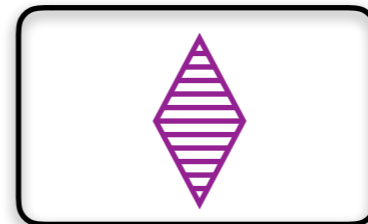
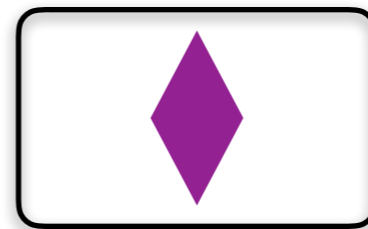
**Color**



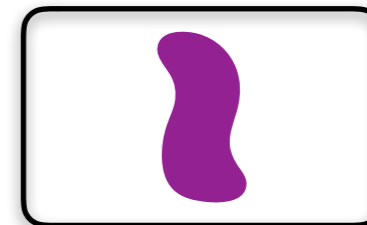
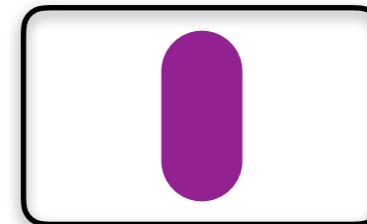
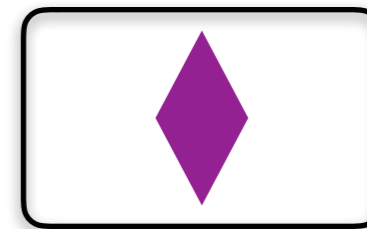
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**Shading**

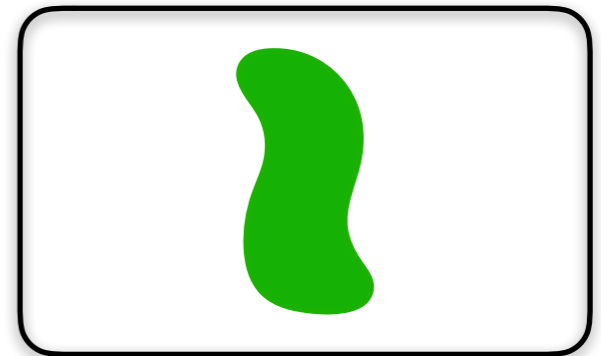
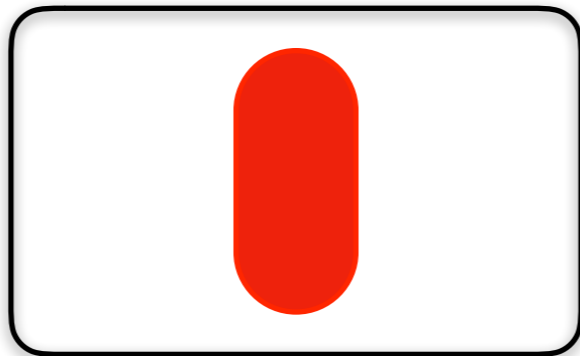
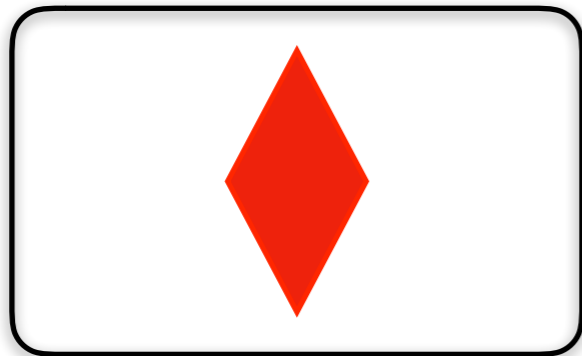


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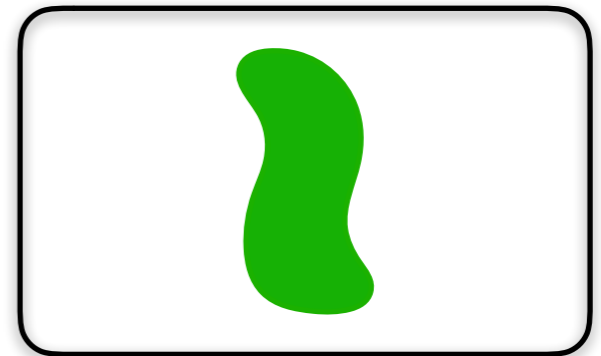
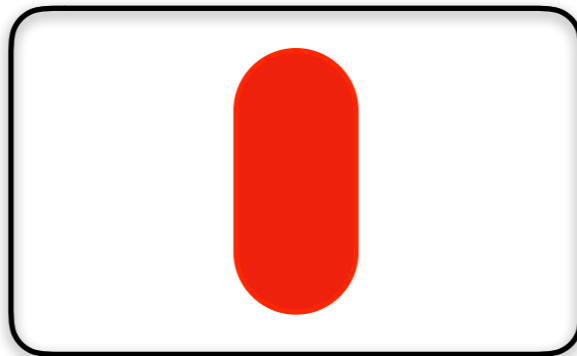
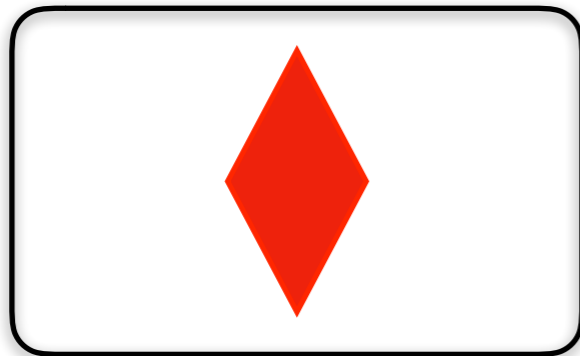


**For every property, all are the same or all are different**



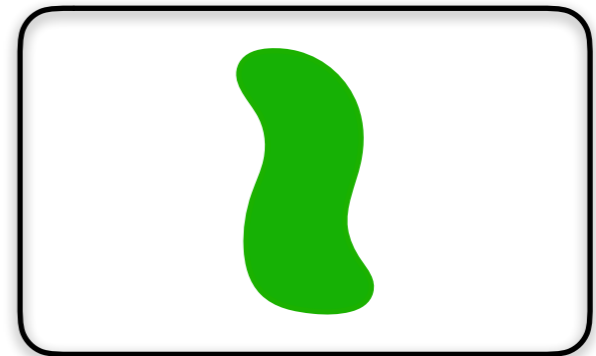
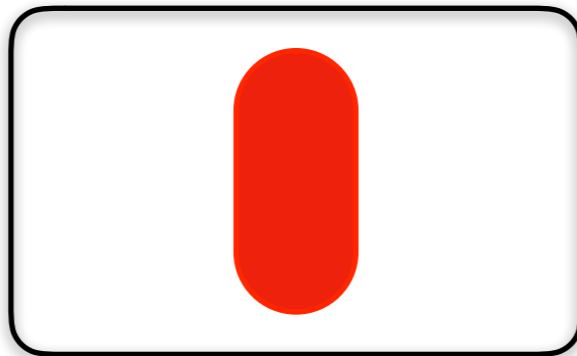
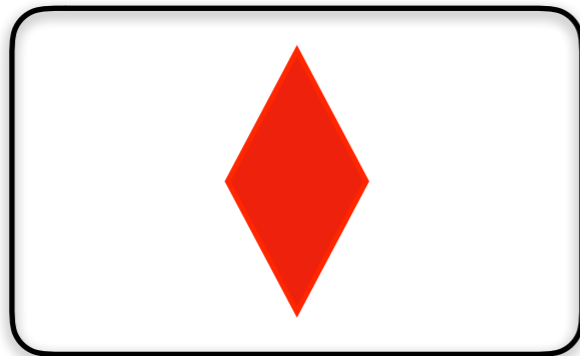
**For every property, all are the same or all are different**

**Two are red, one is green**



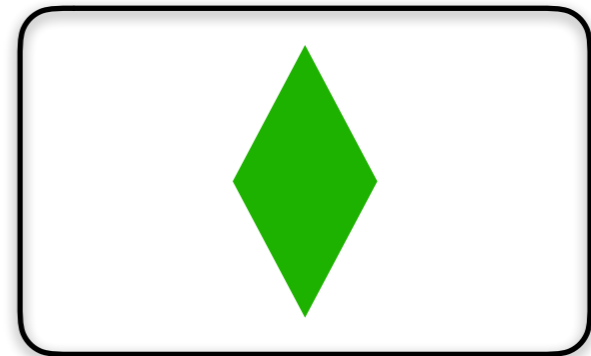
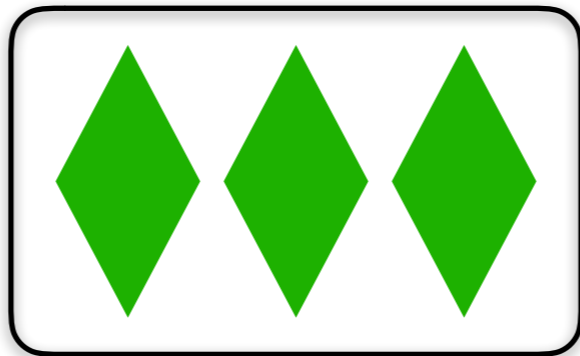
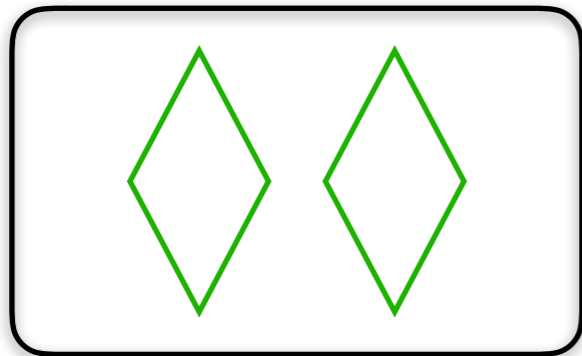
**For every property, all are the same or all are different**

**Two are red, one is green**



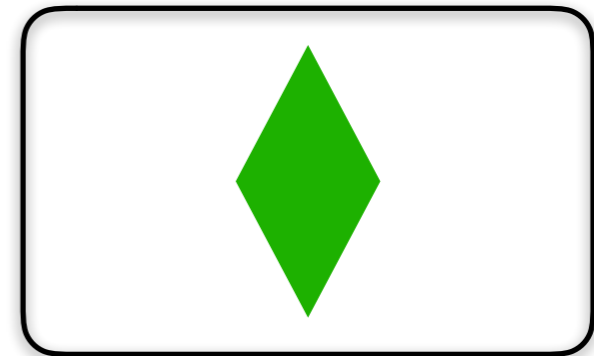
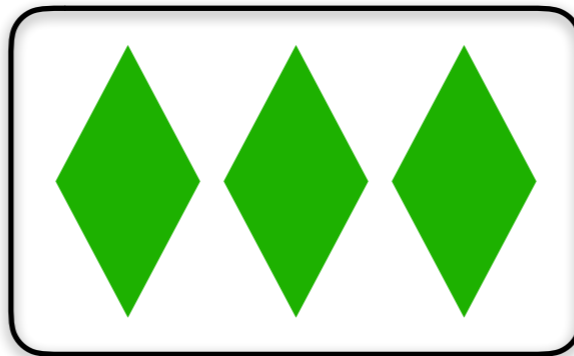
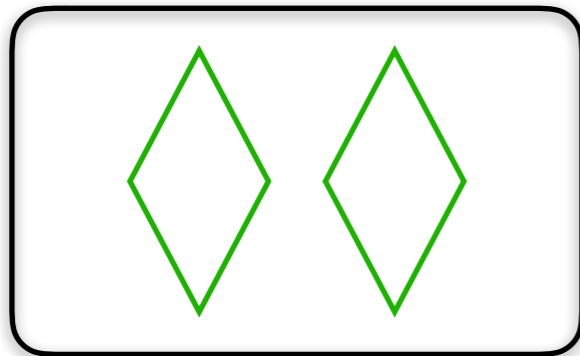
**NOT A SET!**

**For every property, all are the same or all are different**



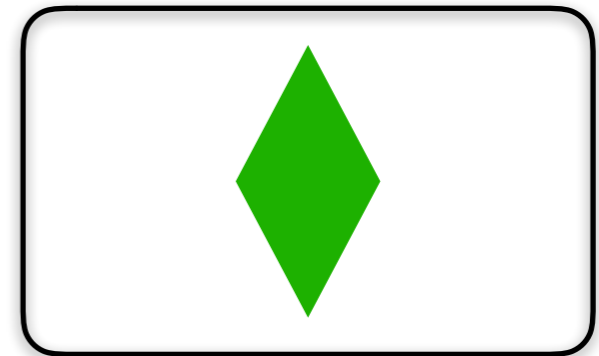
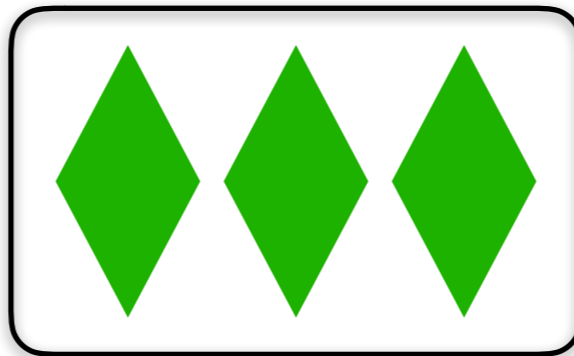
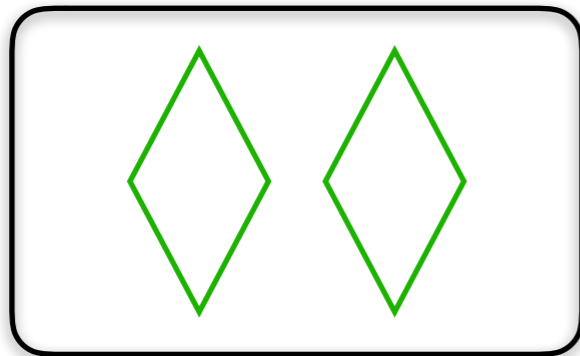
**For every property, all are the same or all are different**

**Two are solid, one is empty**



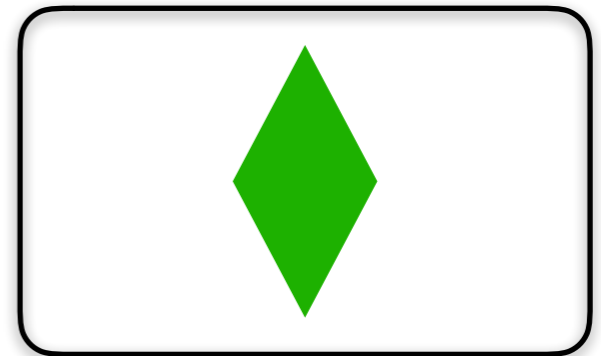
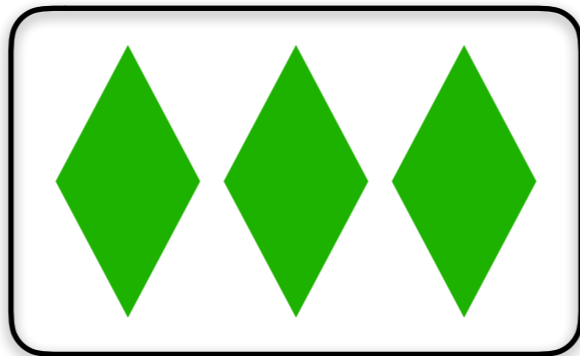
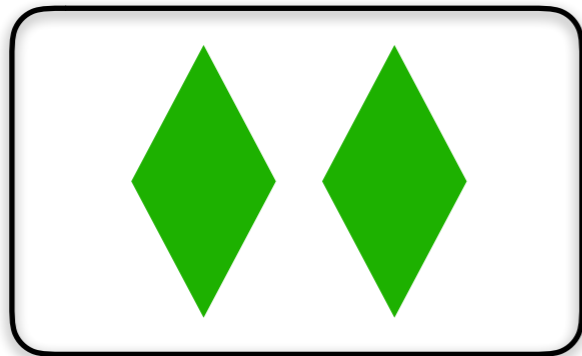
**For every property, all are the same or all are different**

**Two are solid, one is empty**



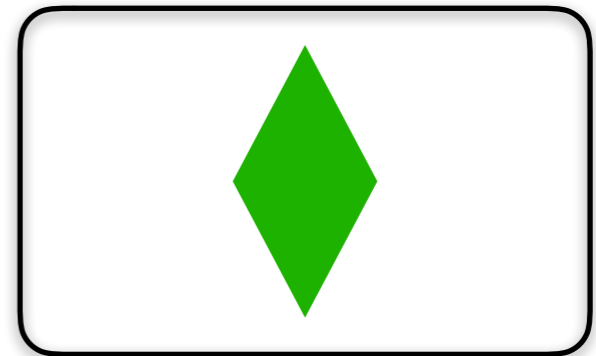
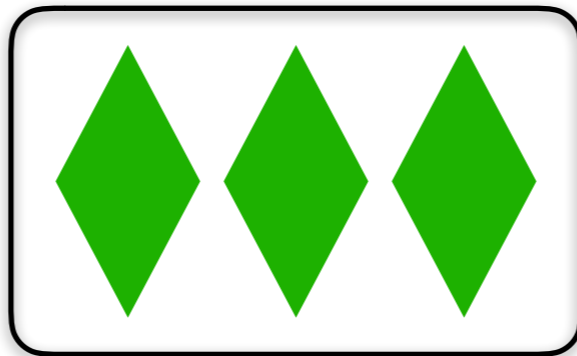
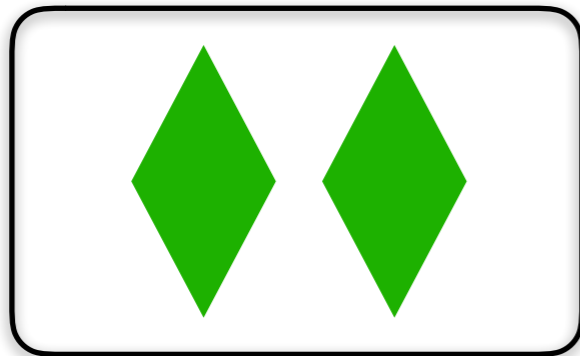
**NOT A SET!**

**For every property, all are the same or all are different**



For every property, all are the same or all are different

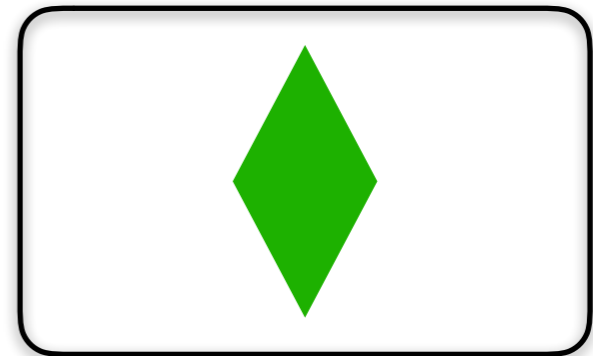
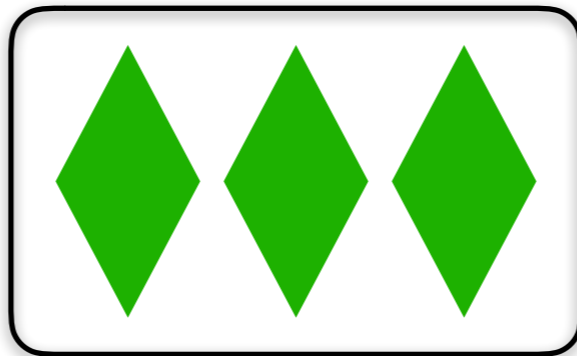
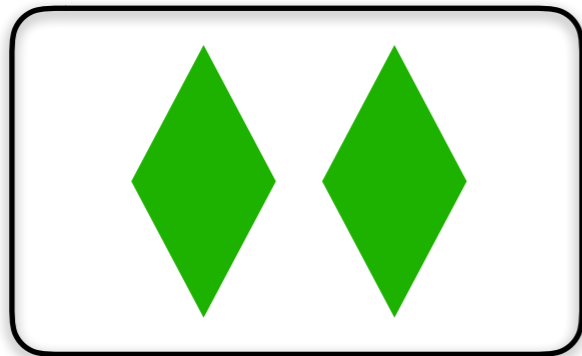
All solid, all green, all diamonds, all *different* numbers





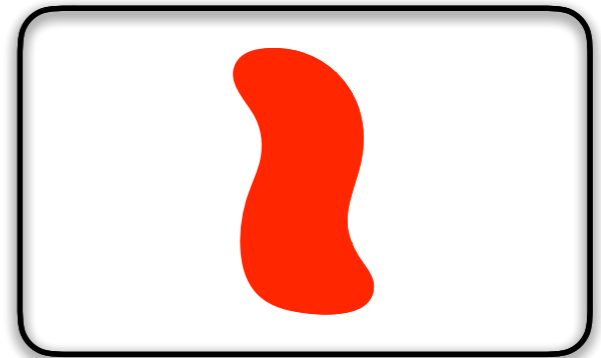
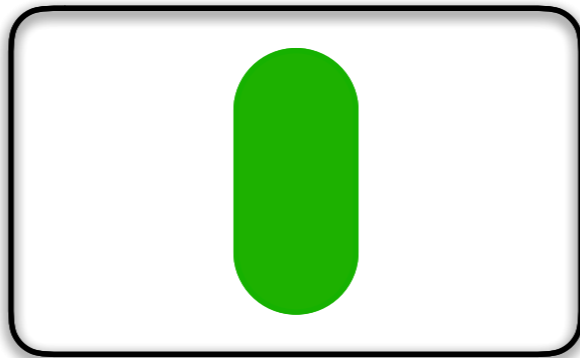
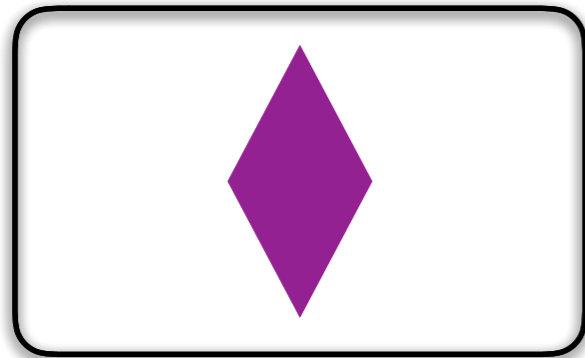
For every property, all are the same or all are different

All solid, all green, all diamonds, all *different* numbers



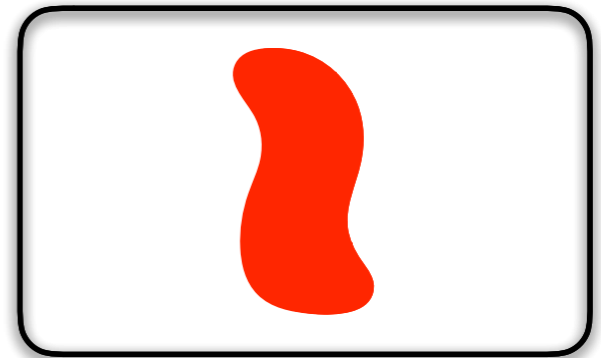
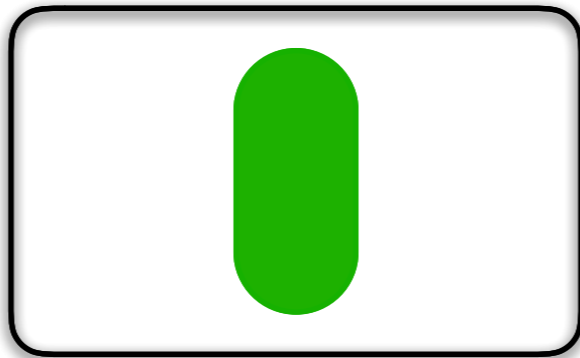
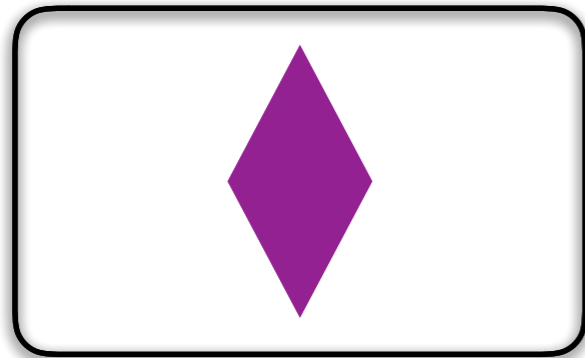
**A SET!!**

**For every property, all are the same or all are different**



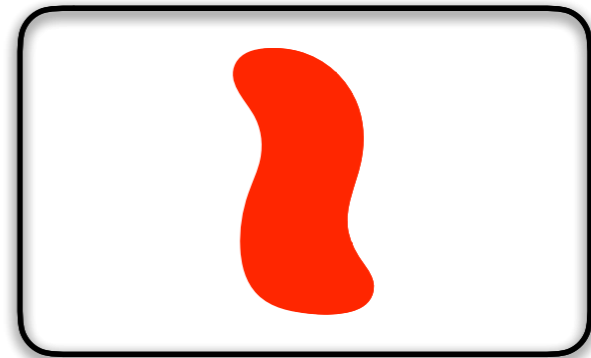
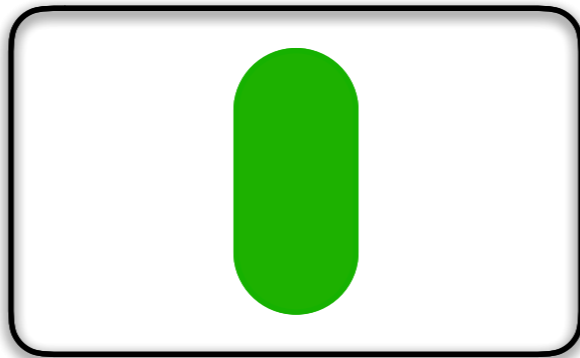
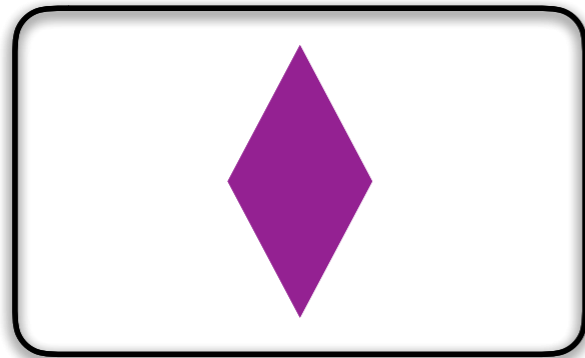
**For every property, all are the same or all are different**

**All the same number, all solid,  
all different shapes, all different colors**



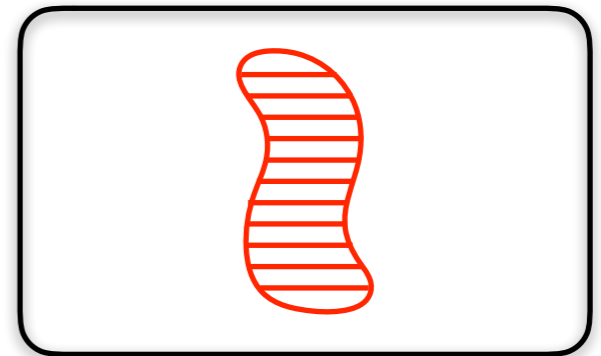
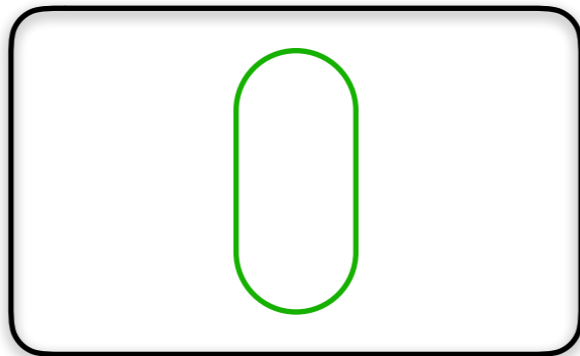
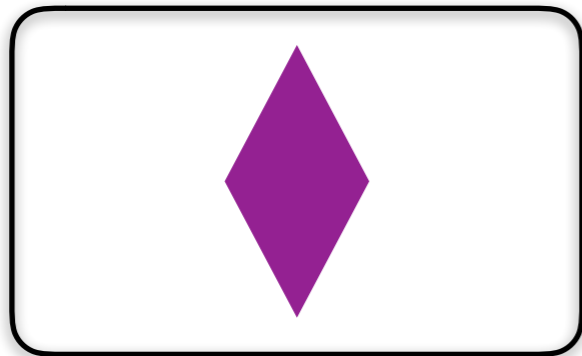
**For every property, all are the  
same or all are different**

**All the same number, all solid,  
all different shapes, all different colors**



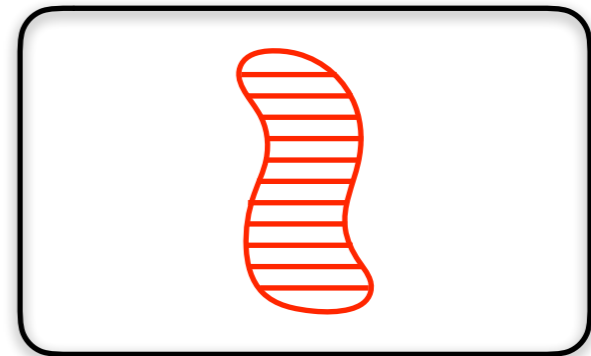
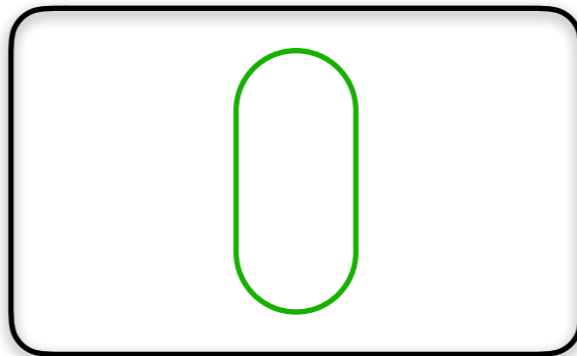
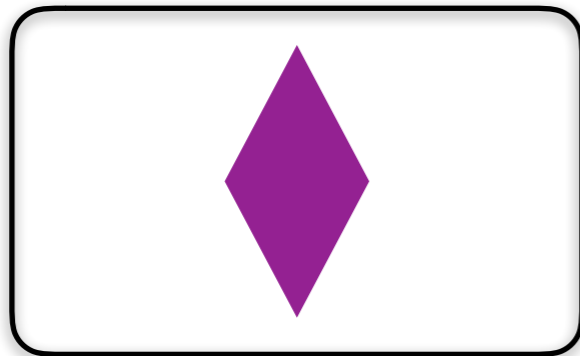
**A SET!!**

**For every property, all are the same or all are different**



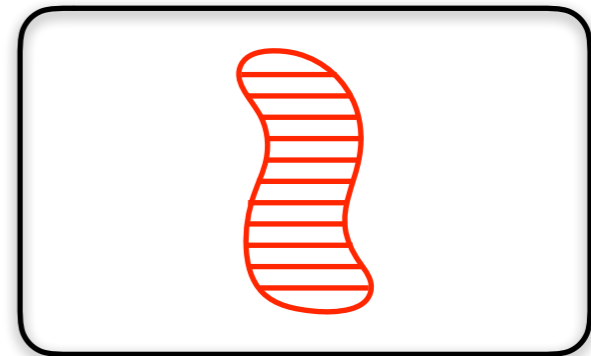
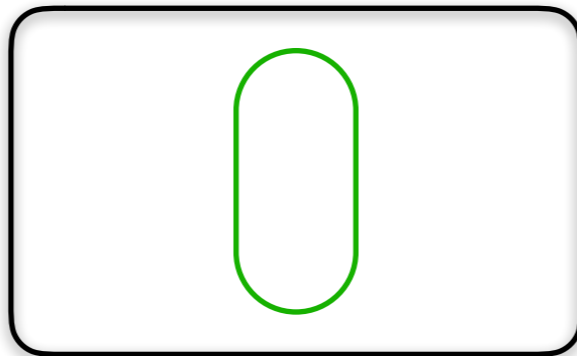
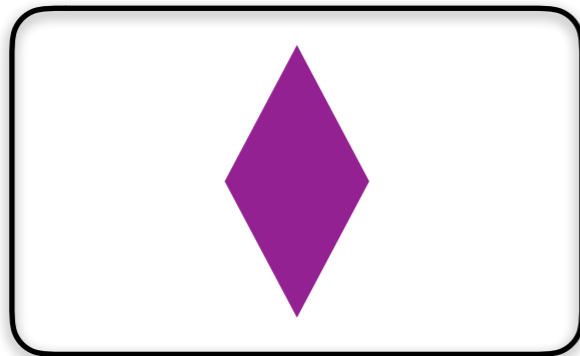
**For every property, all are the same or all are different**

**Same number, all different shading,  
all different shapes, all different colors**



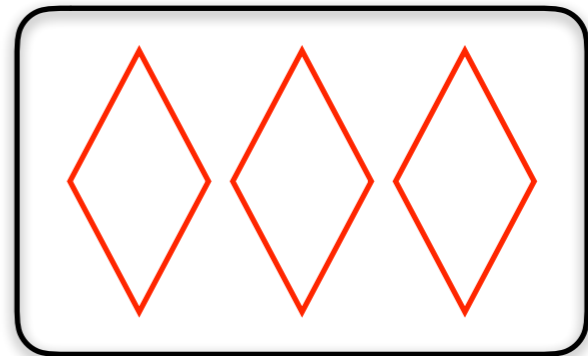
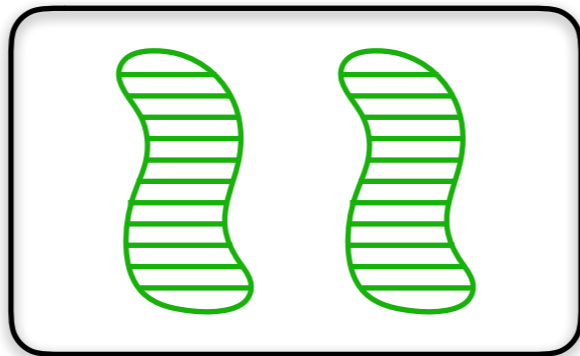
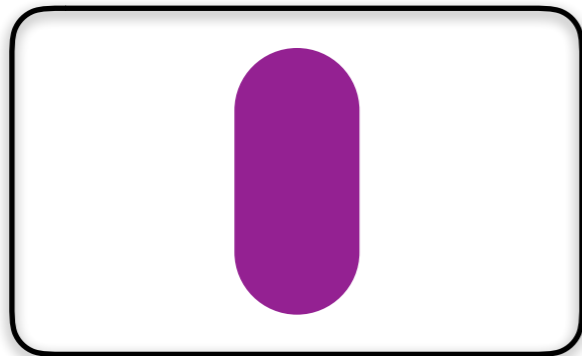
**For every property, all are the same or all are different**

**Same number, all different shading,  
all different shapes, all different colors**



**A SET!!**

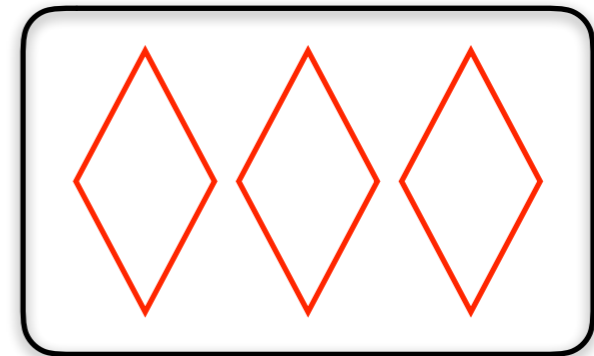
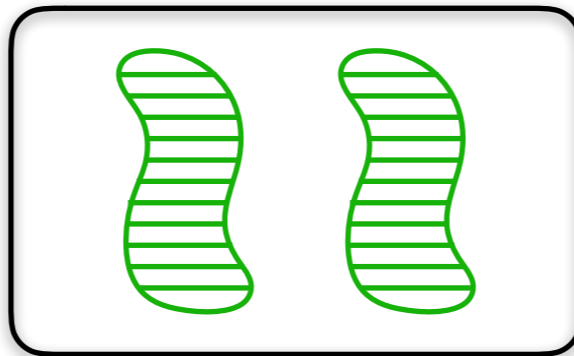
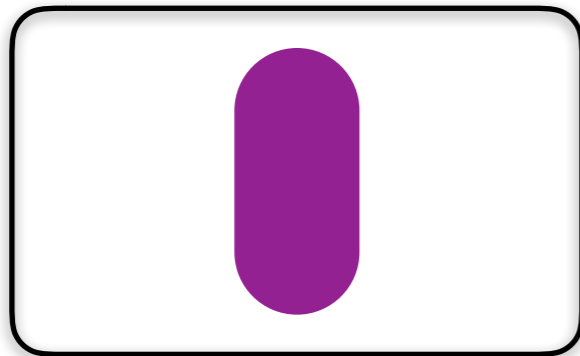
**For every property, all are the same or all are different**





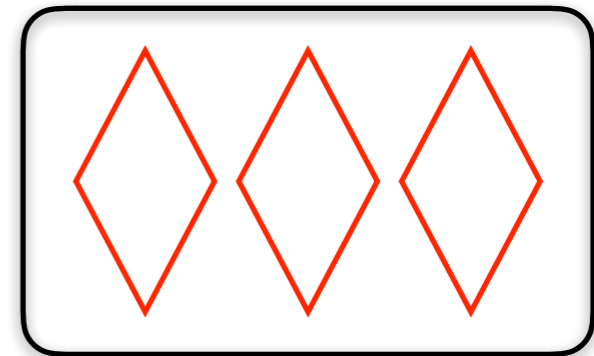
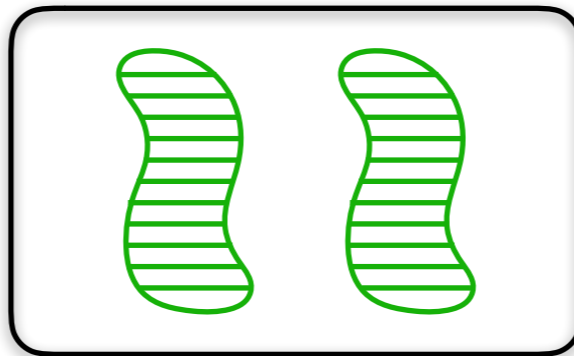
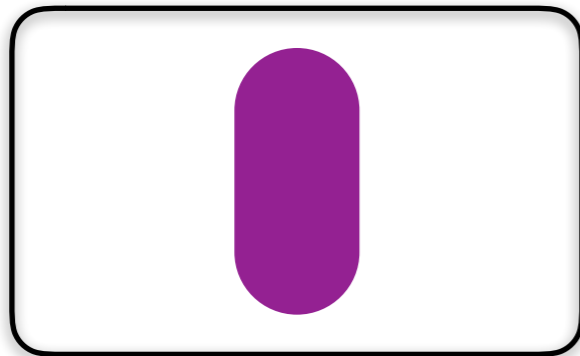
**For every property, all are the same or all are different**

**All different numbers, all different colors,  
all different shapes, all different shading**



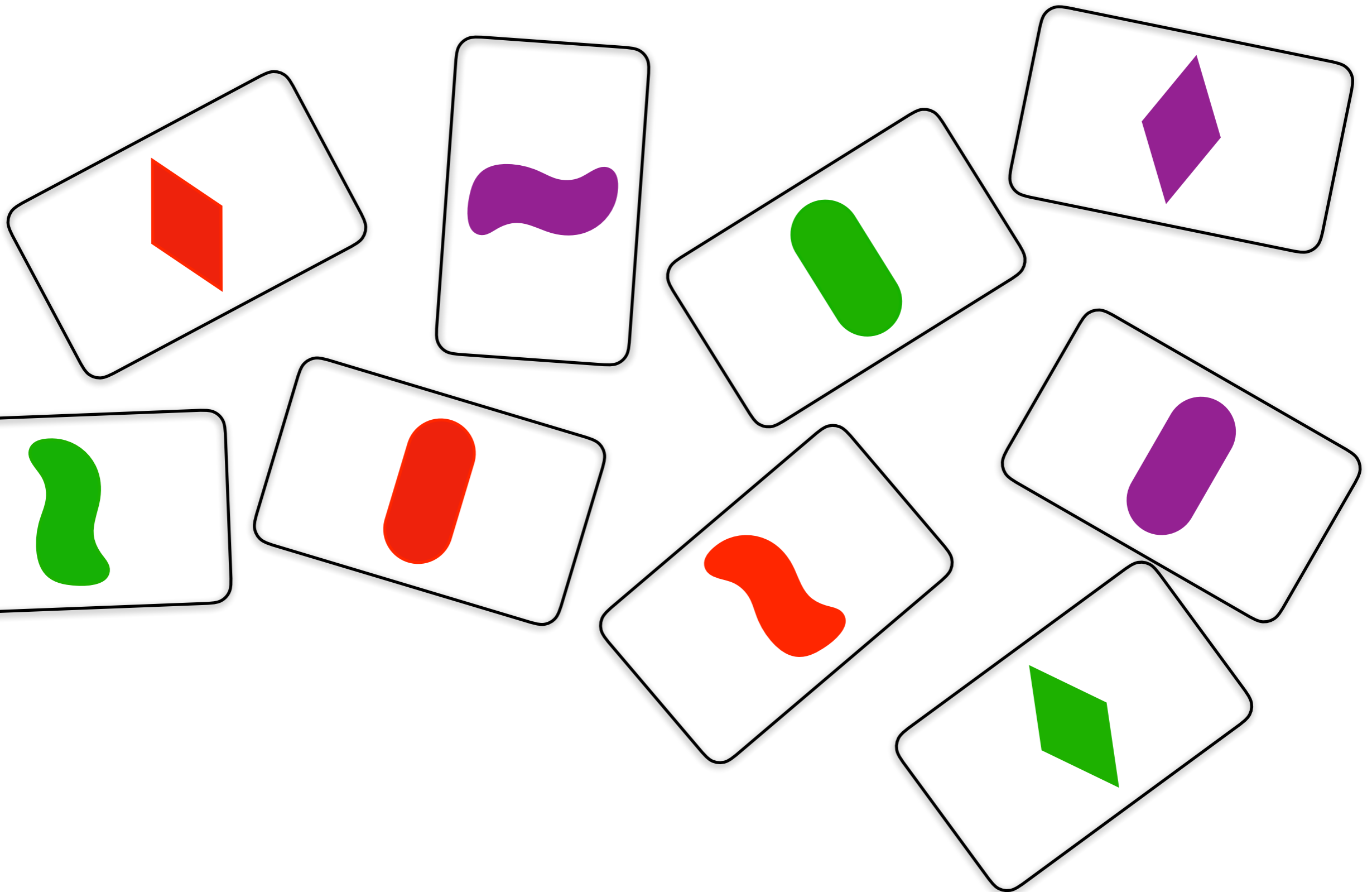
**For every property, all are the same or all are different**

**All different numbers, all different colors,  
all different shapes, all different shading**

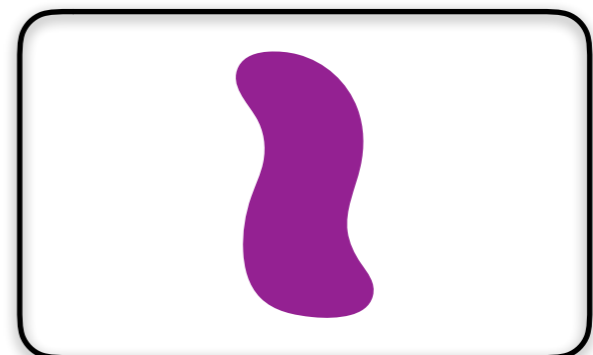
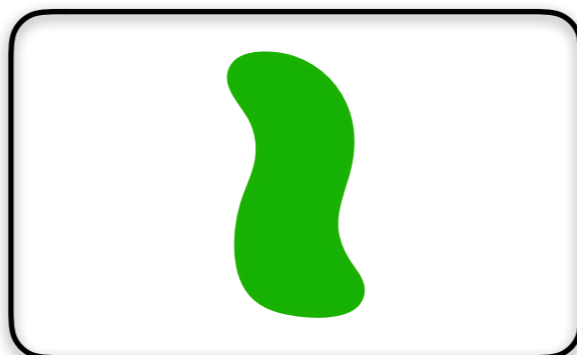
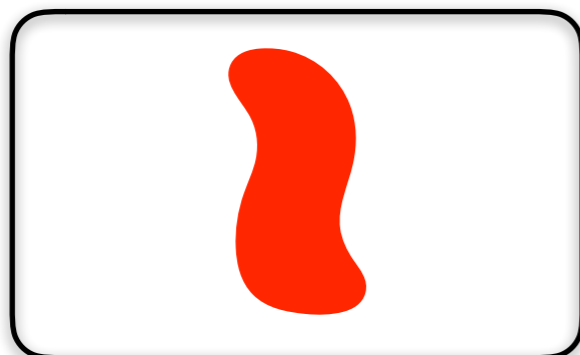
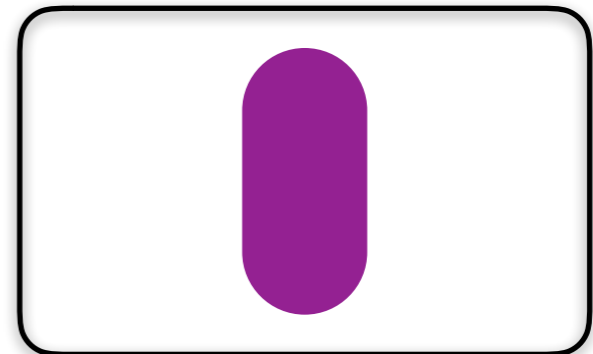
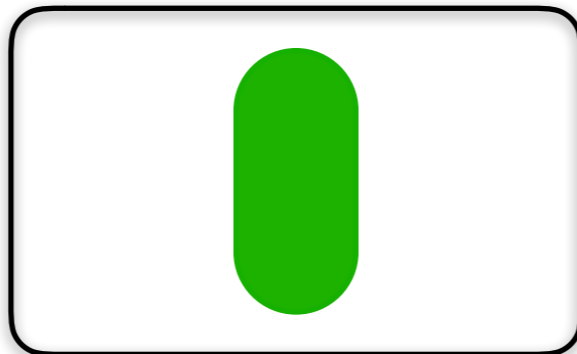
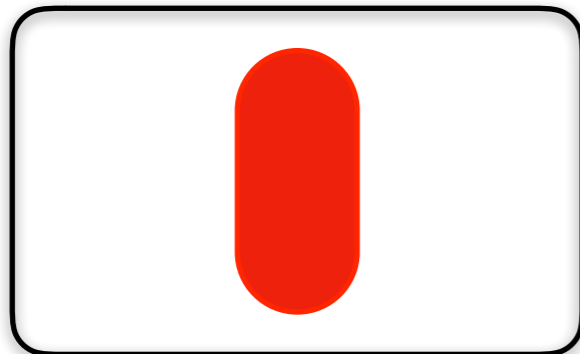
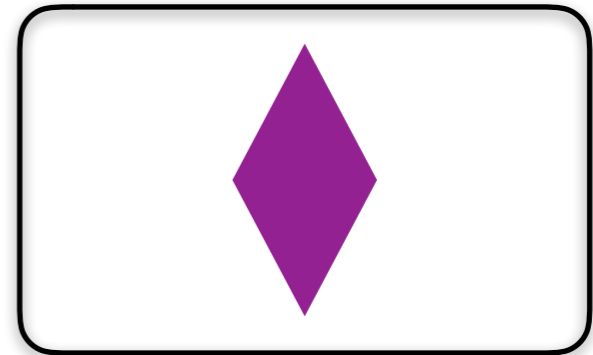
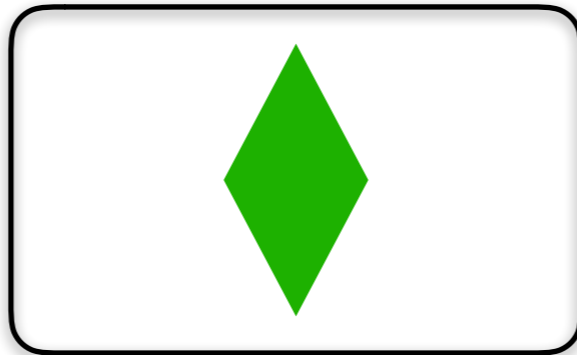
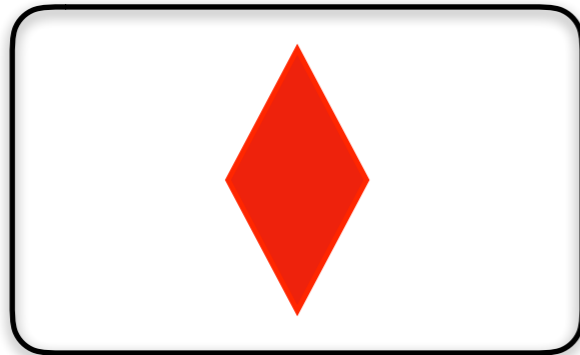


**A SET!!**

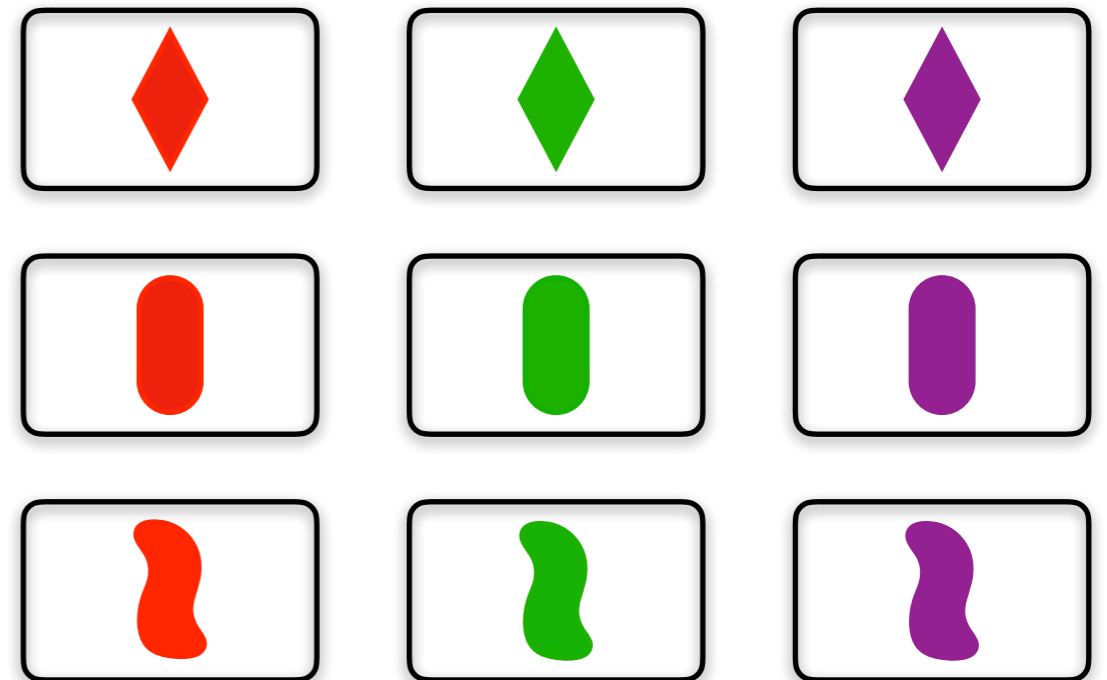
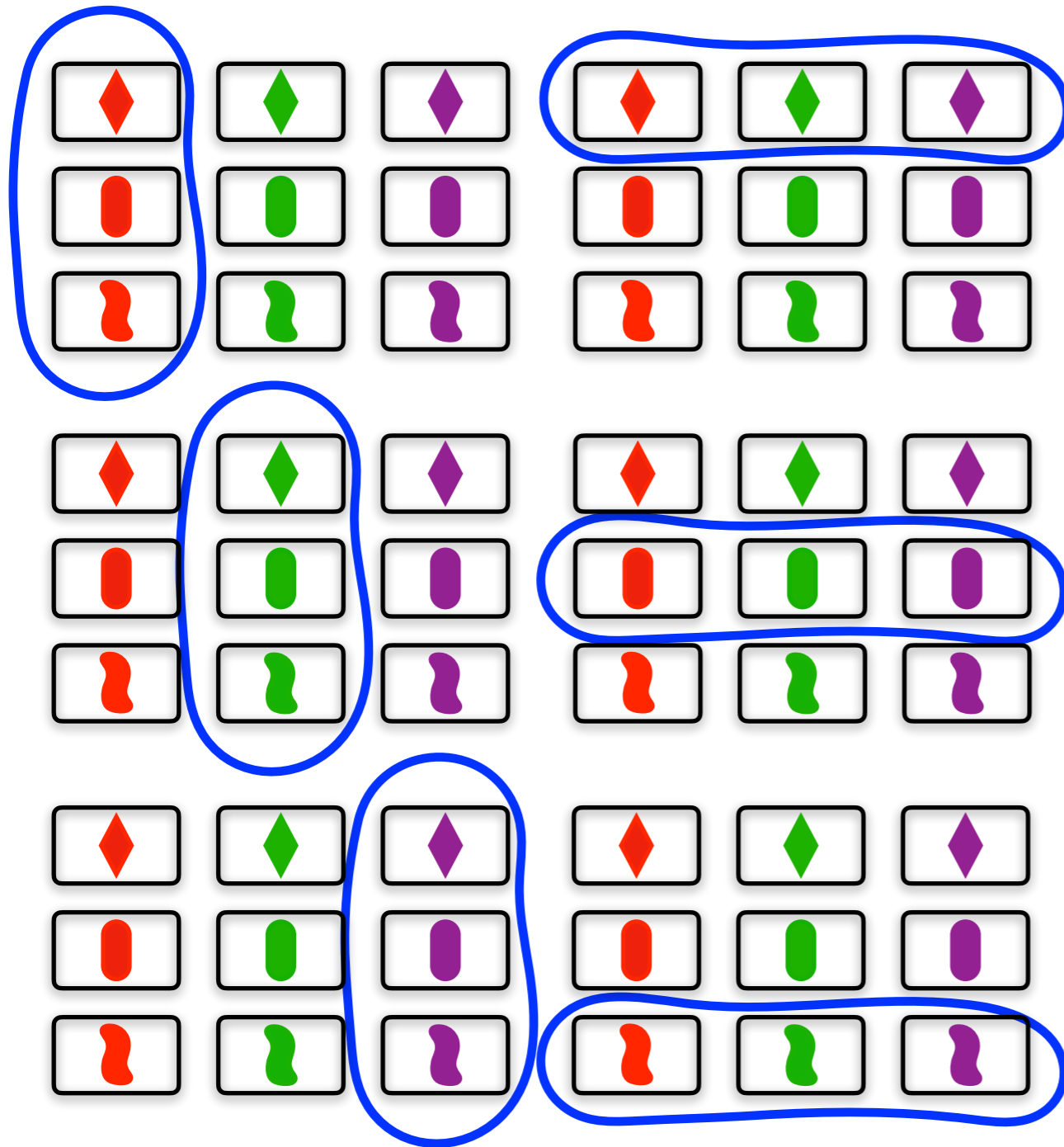
# How many SETs are there?



# What are the possible SETs?

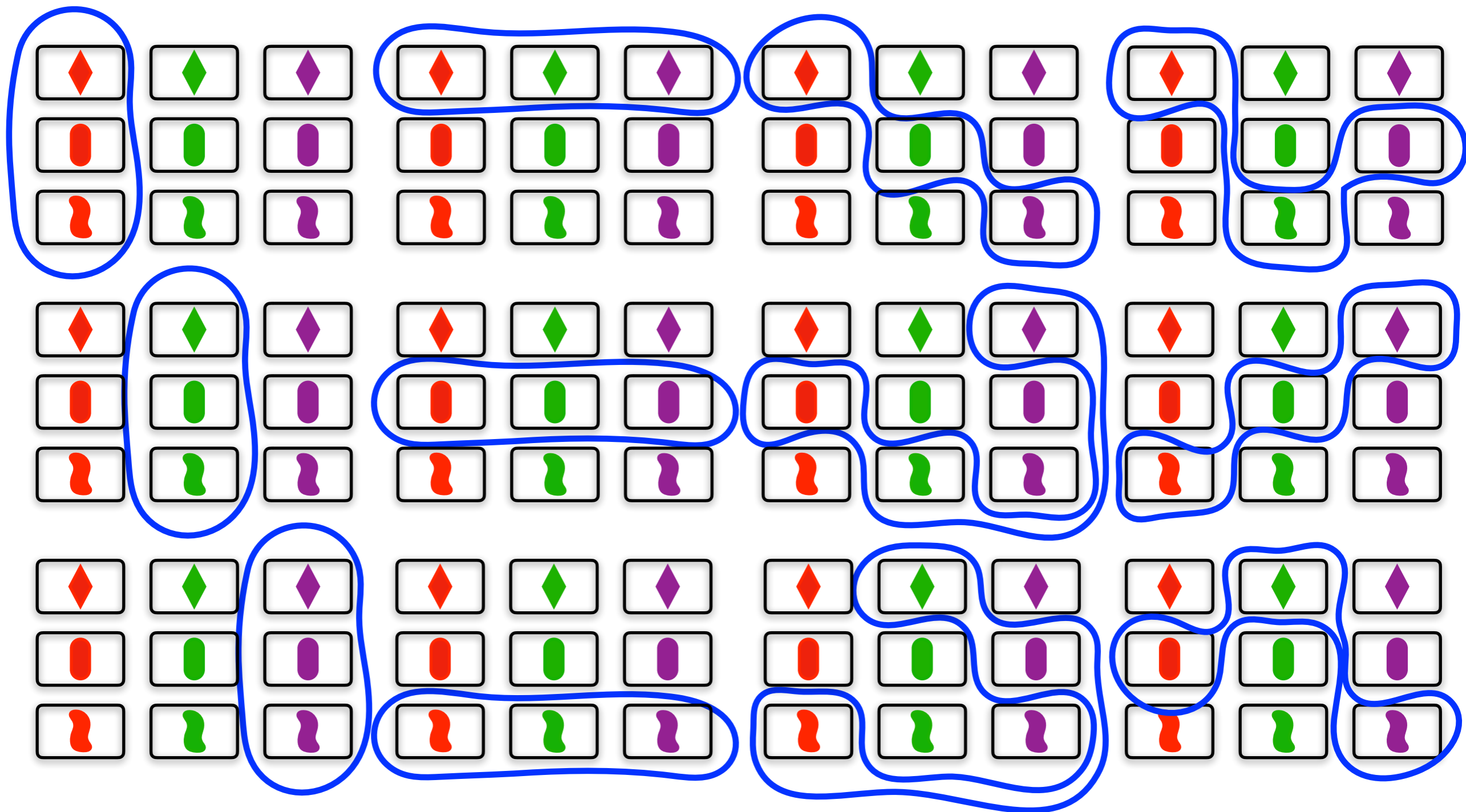


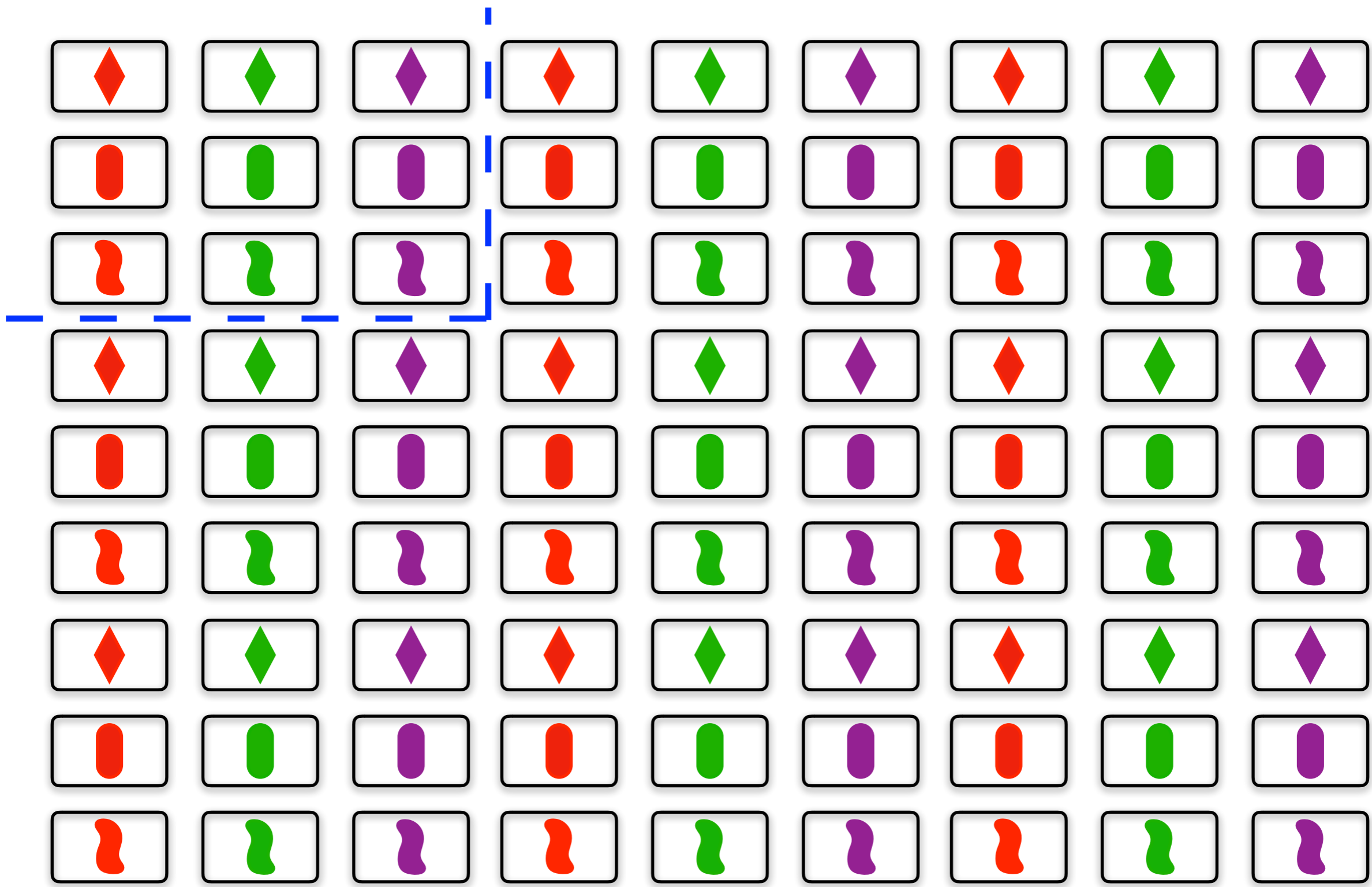
# What are the possible SETs?

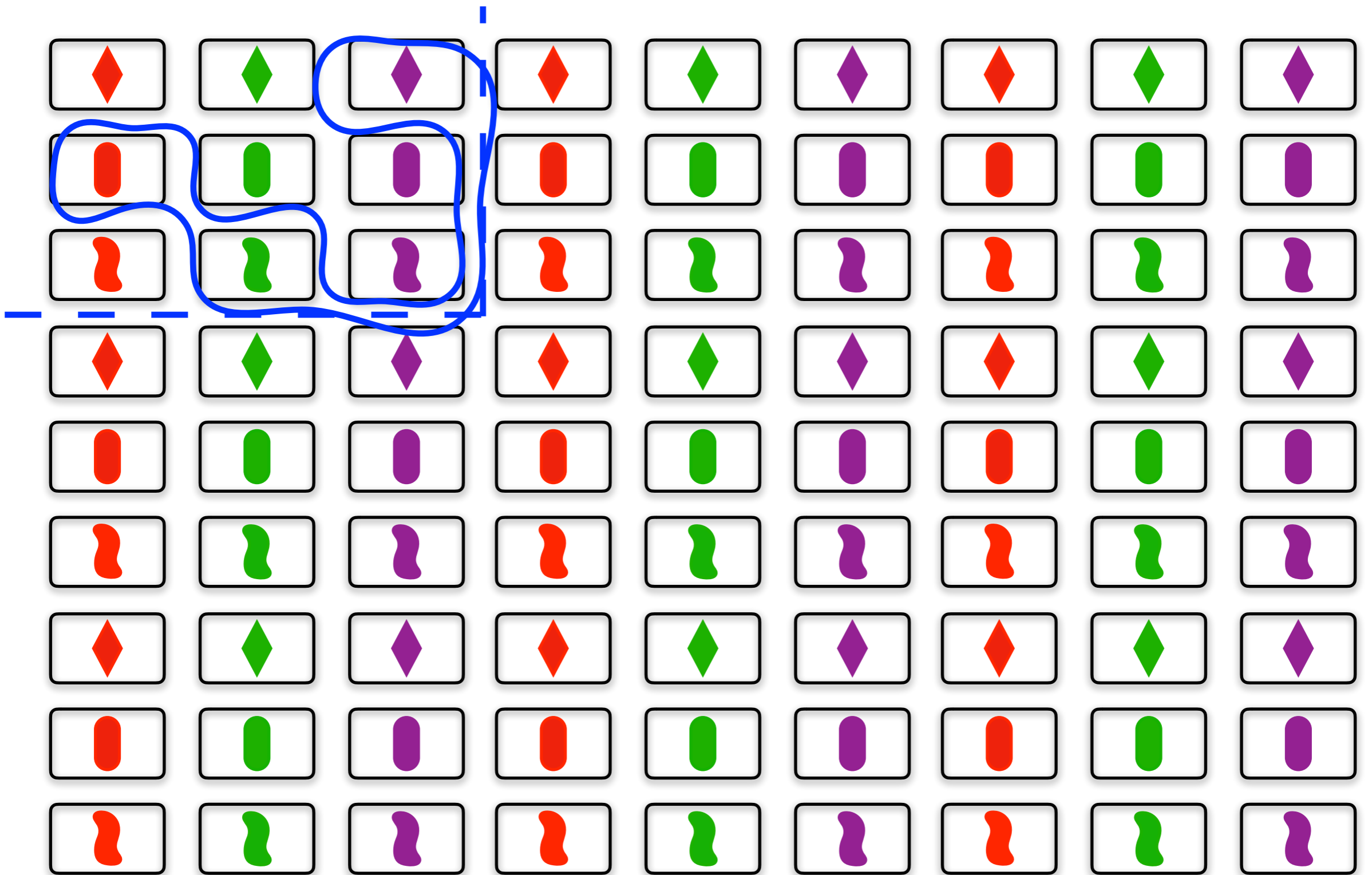


Are there more?

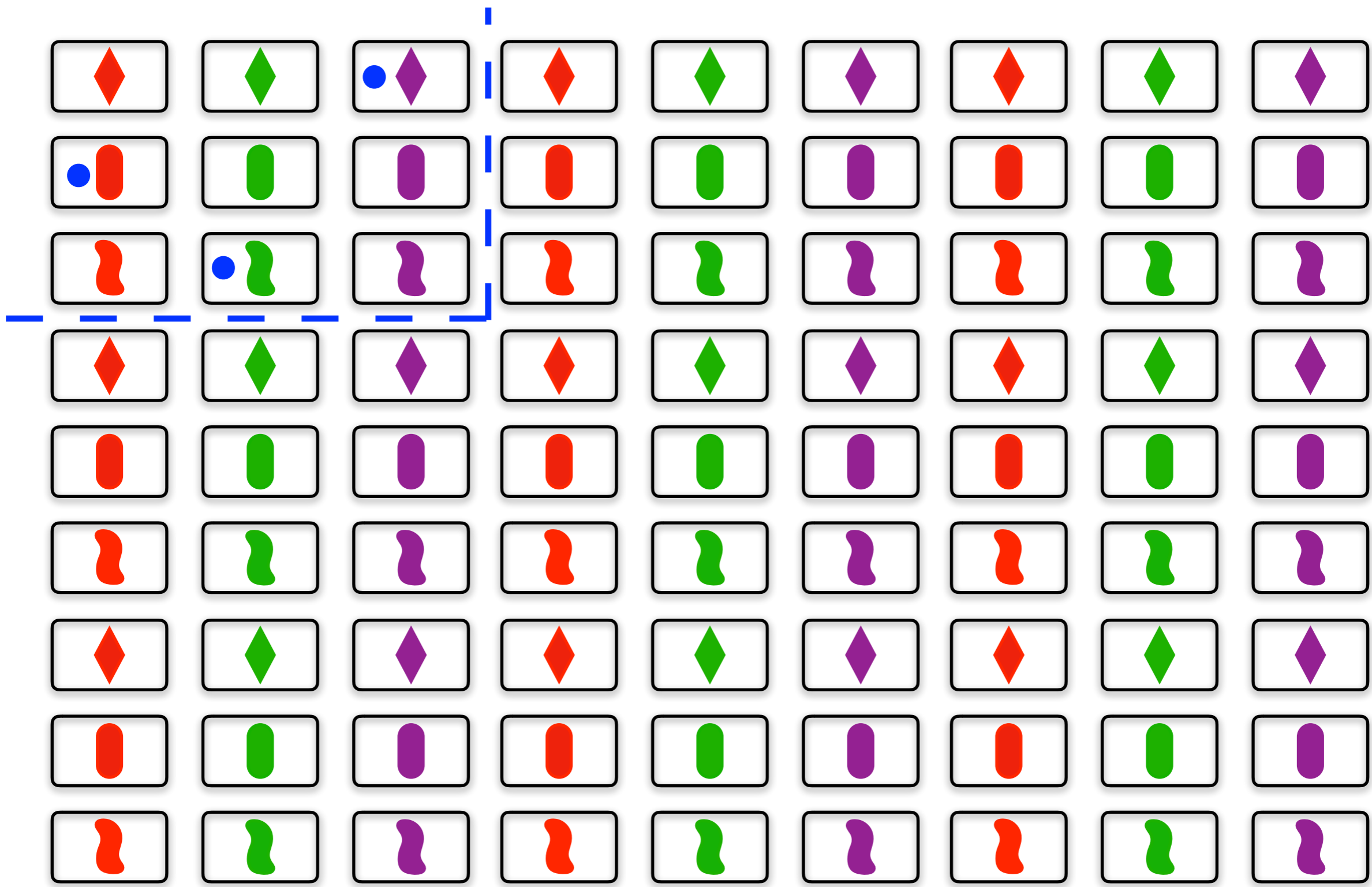
# What are the possible SETs?

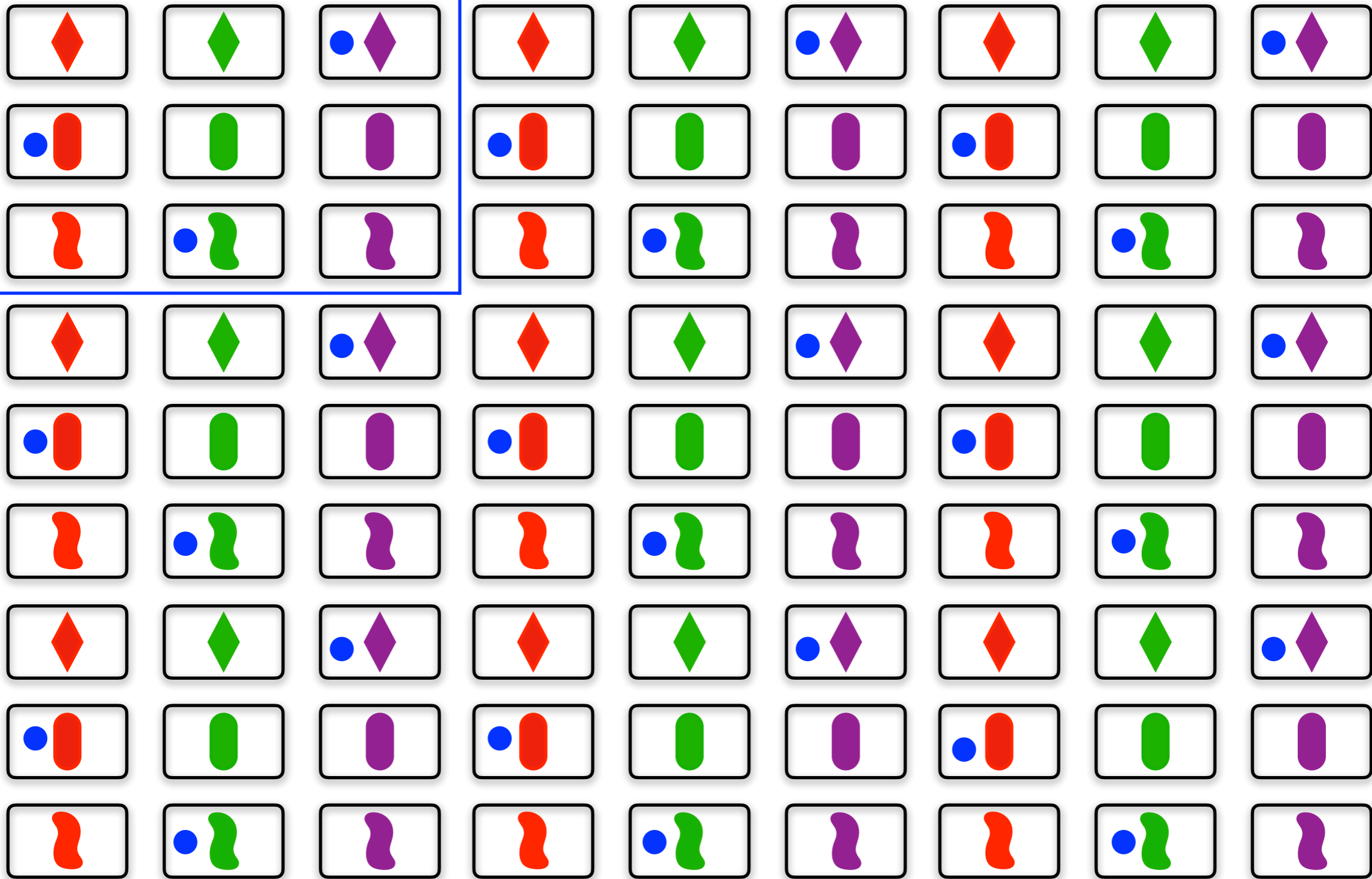


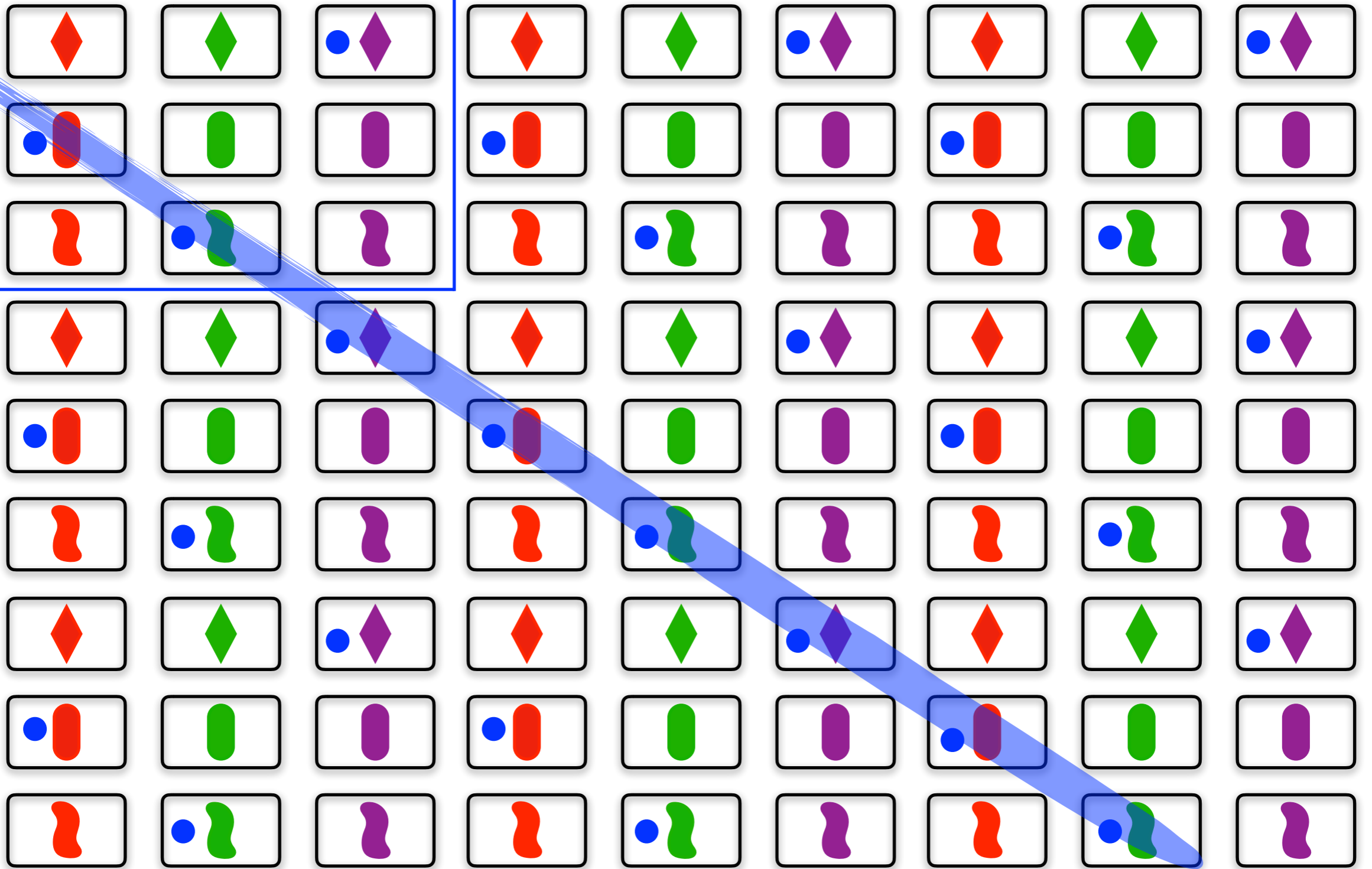




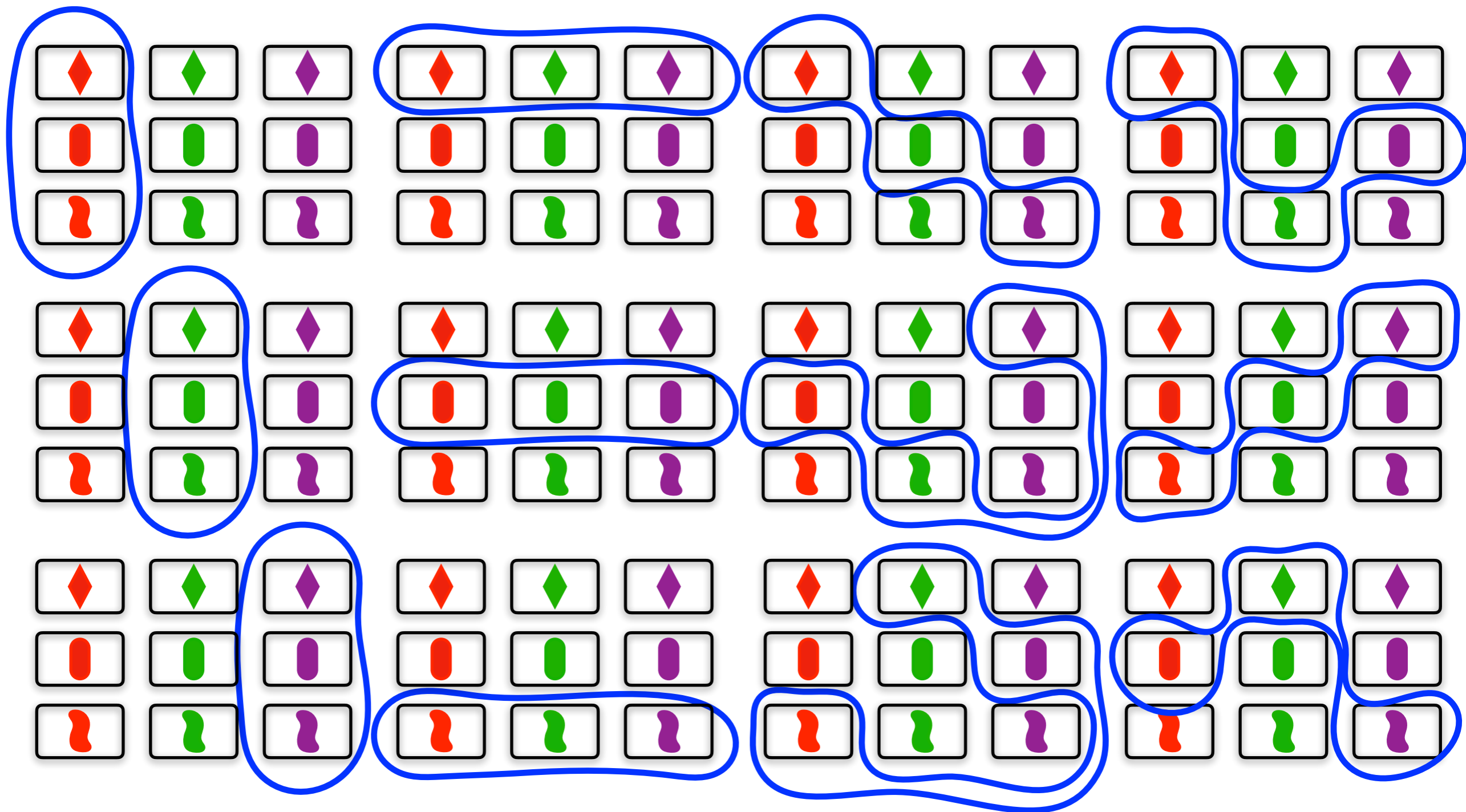




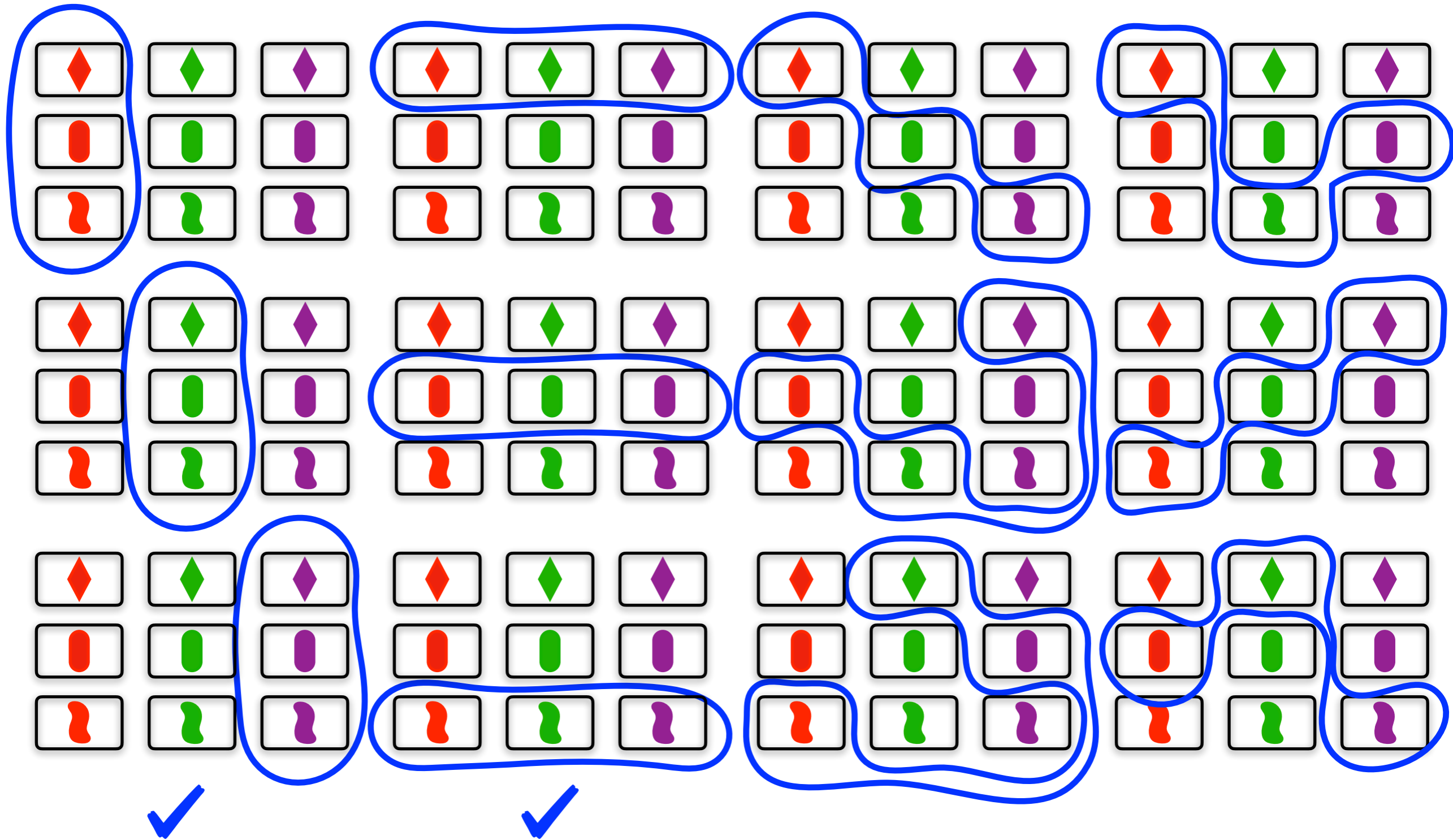




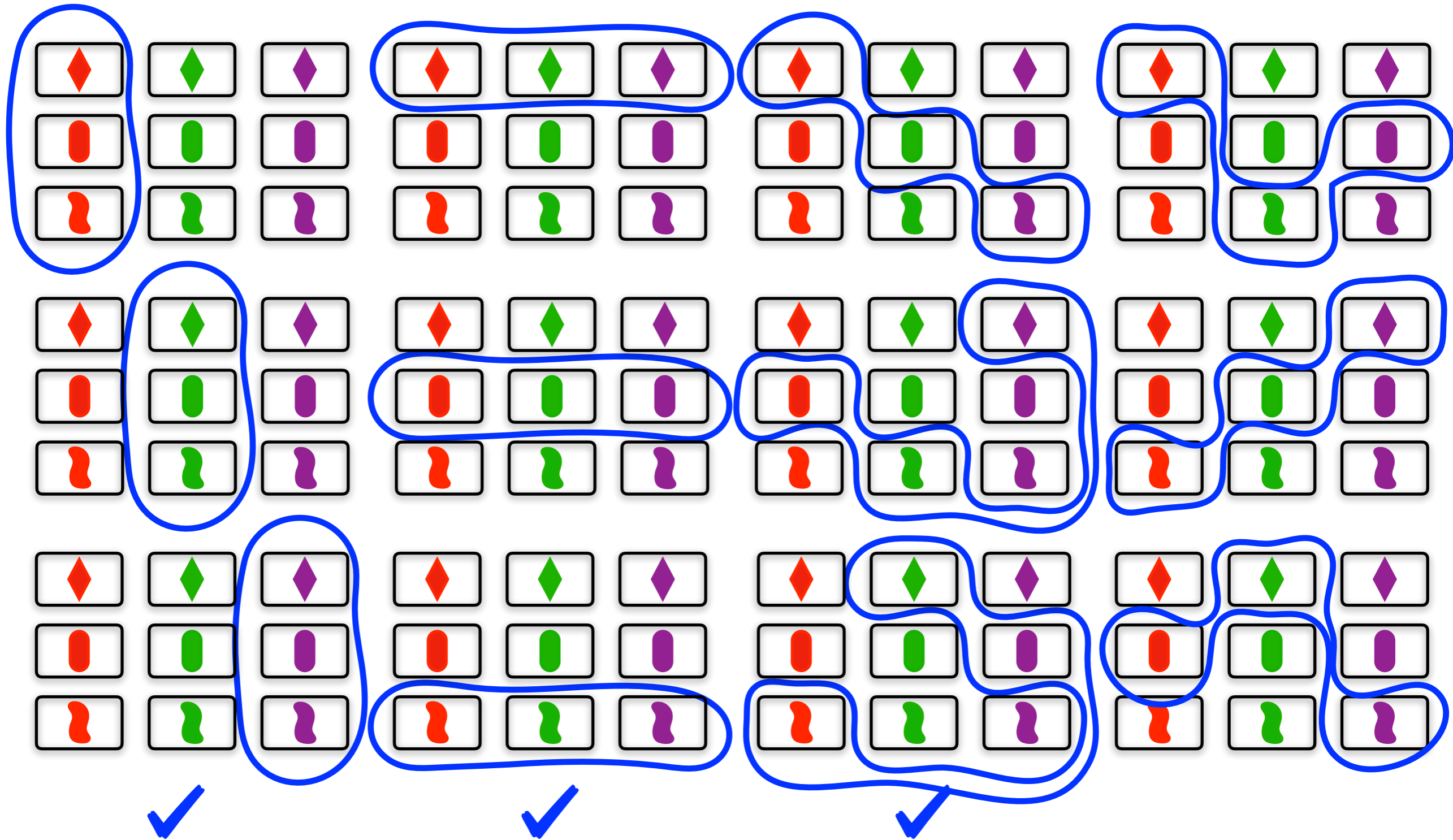
# What are the possible SETs?

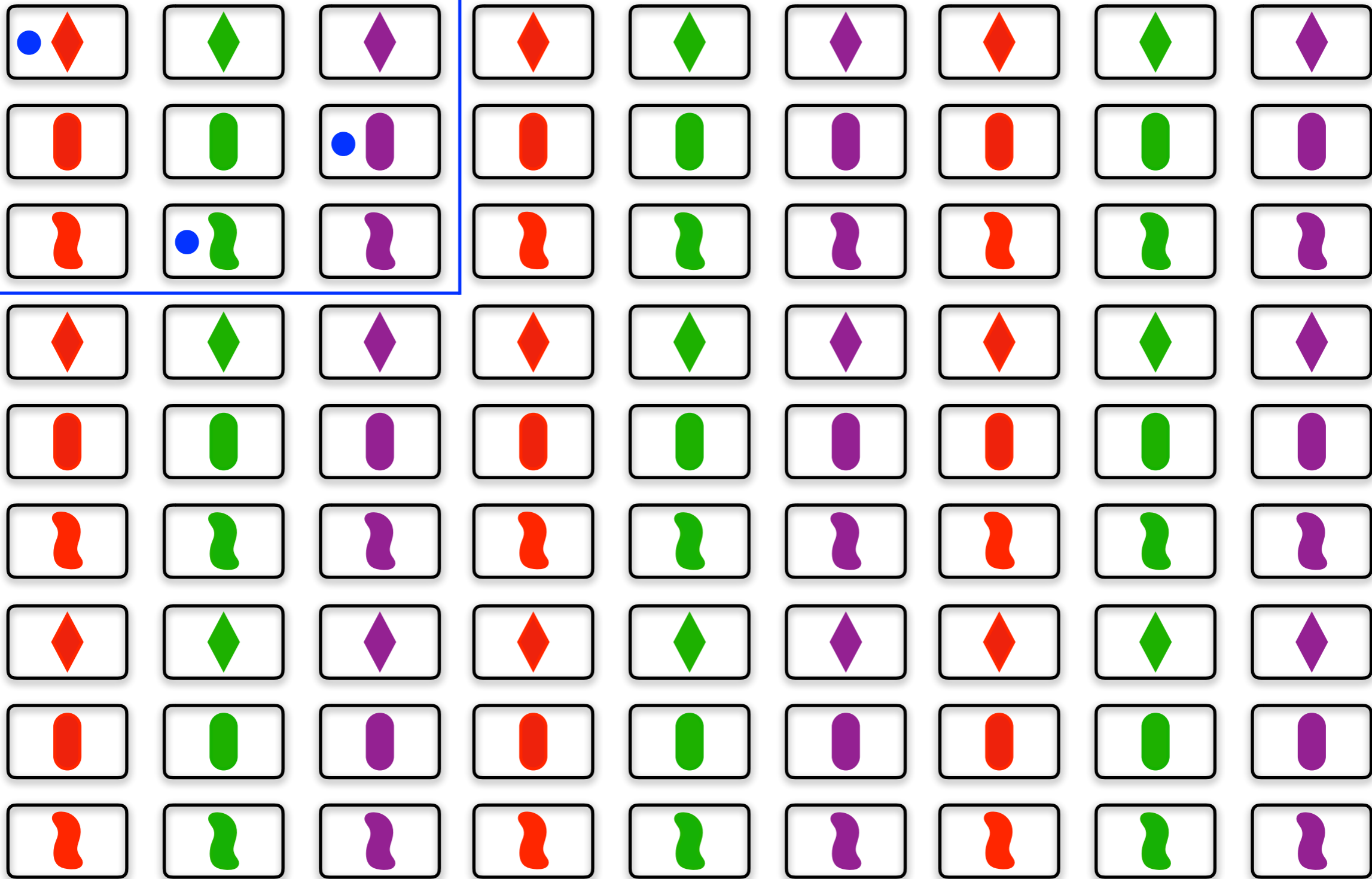


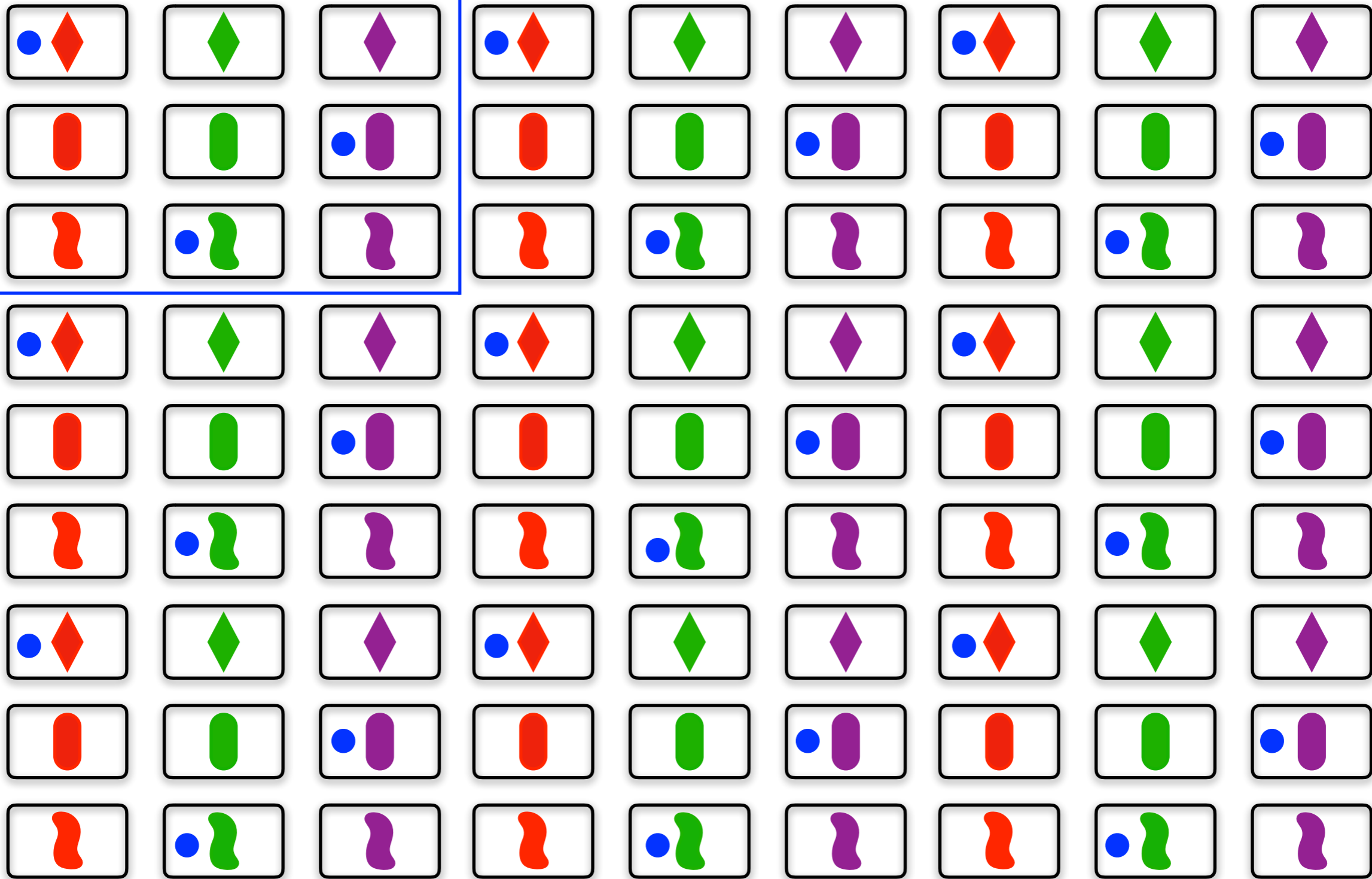
# What are the possible SETs?



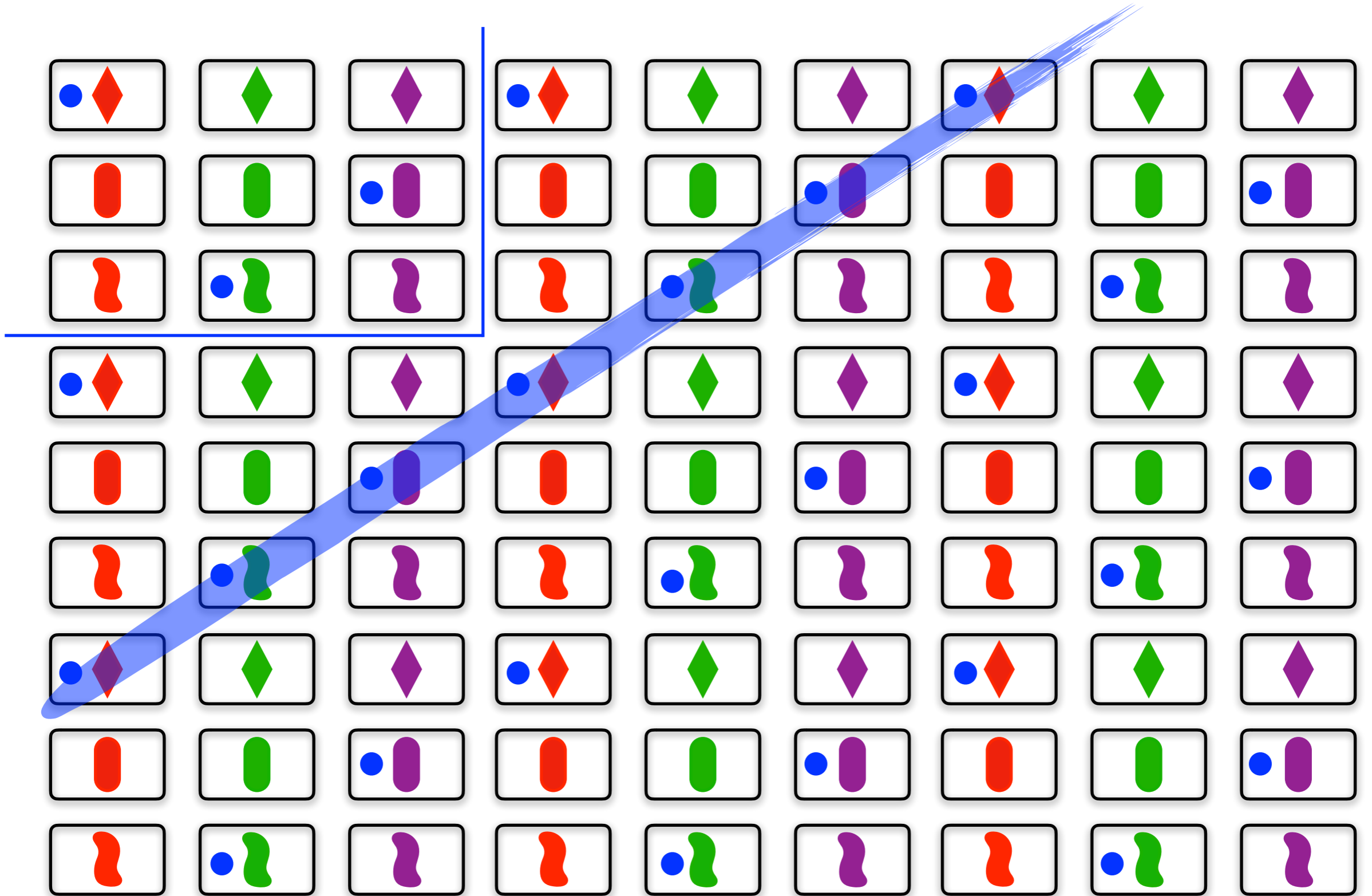
# What are the possible SETs?

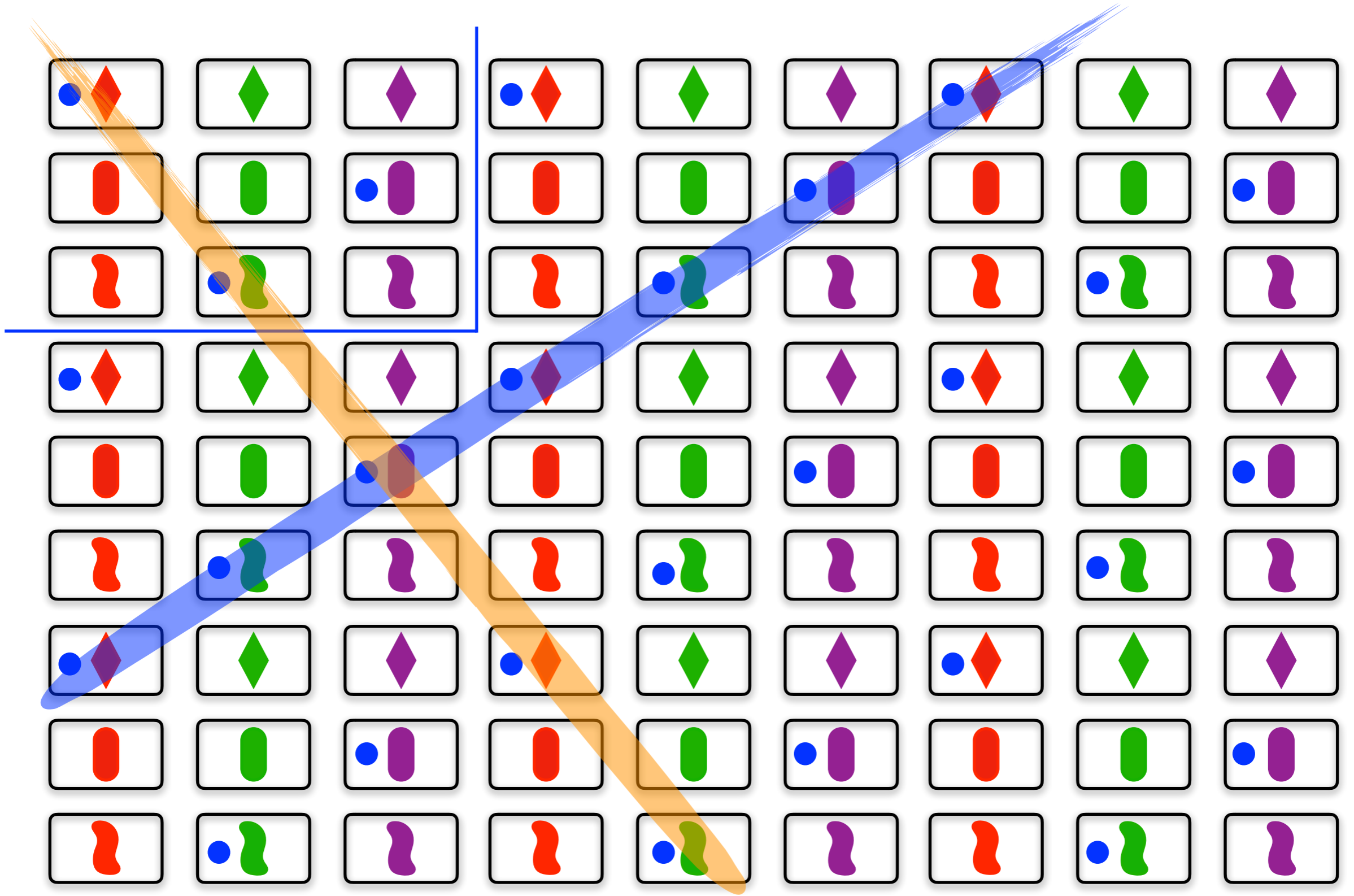


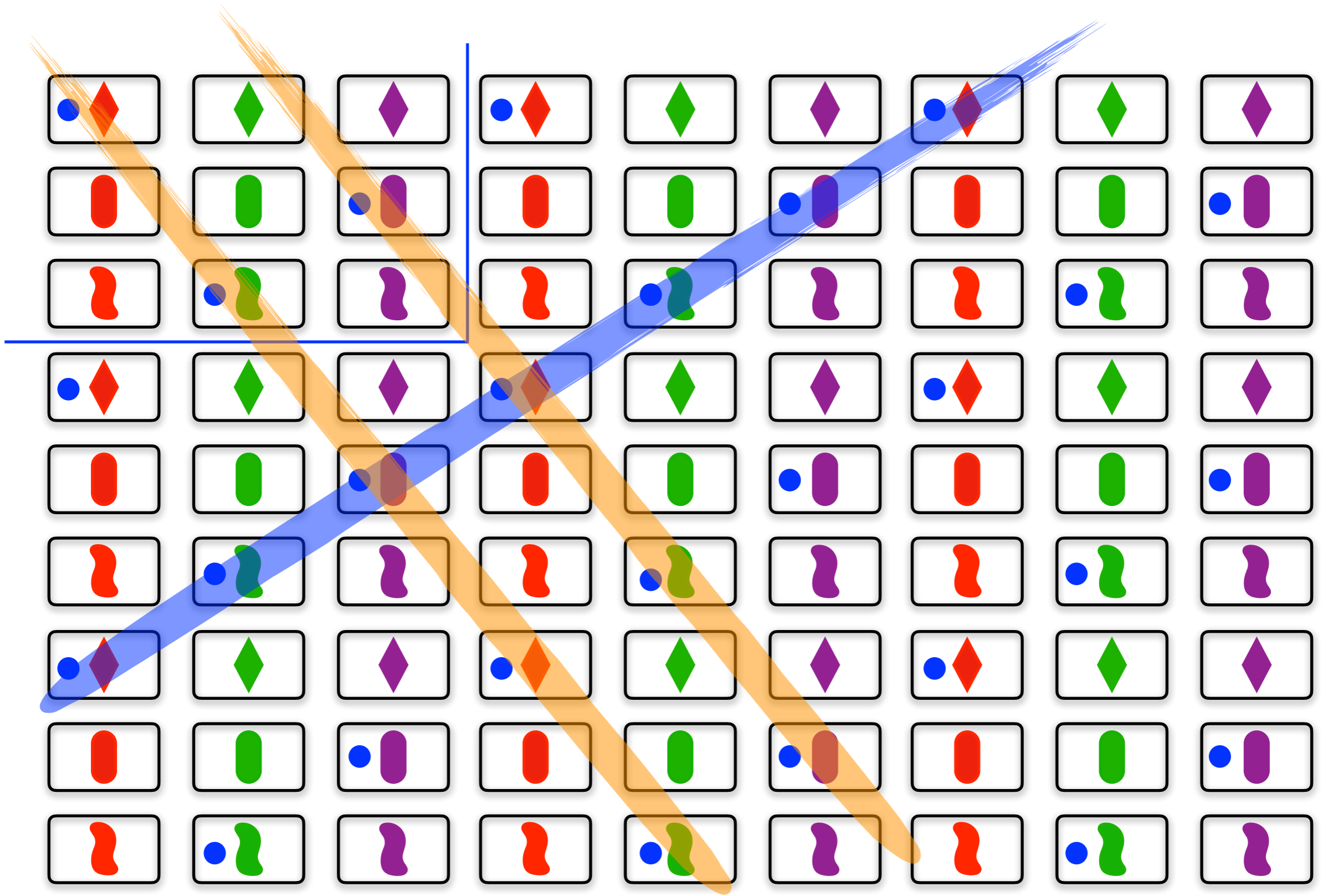




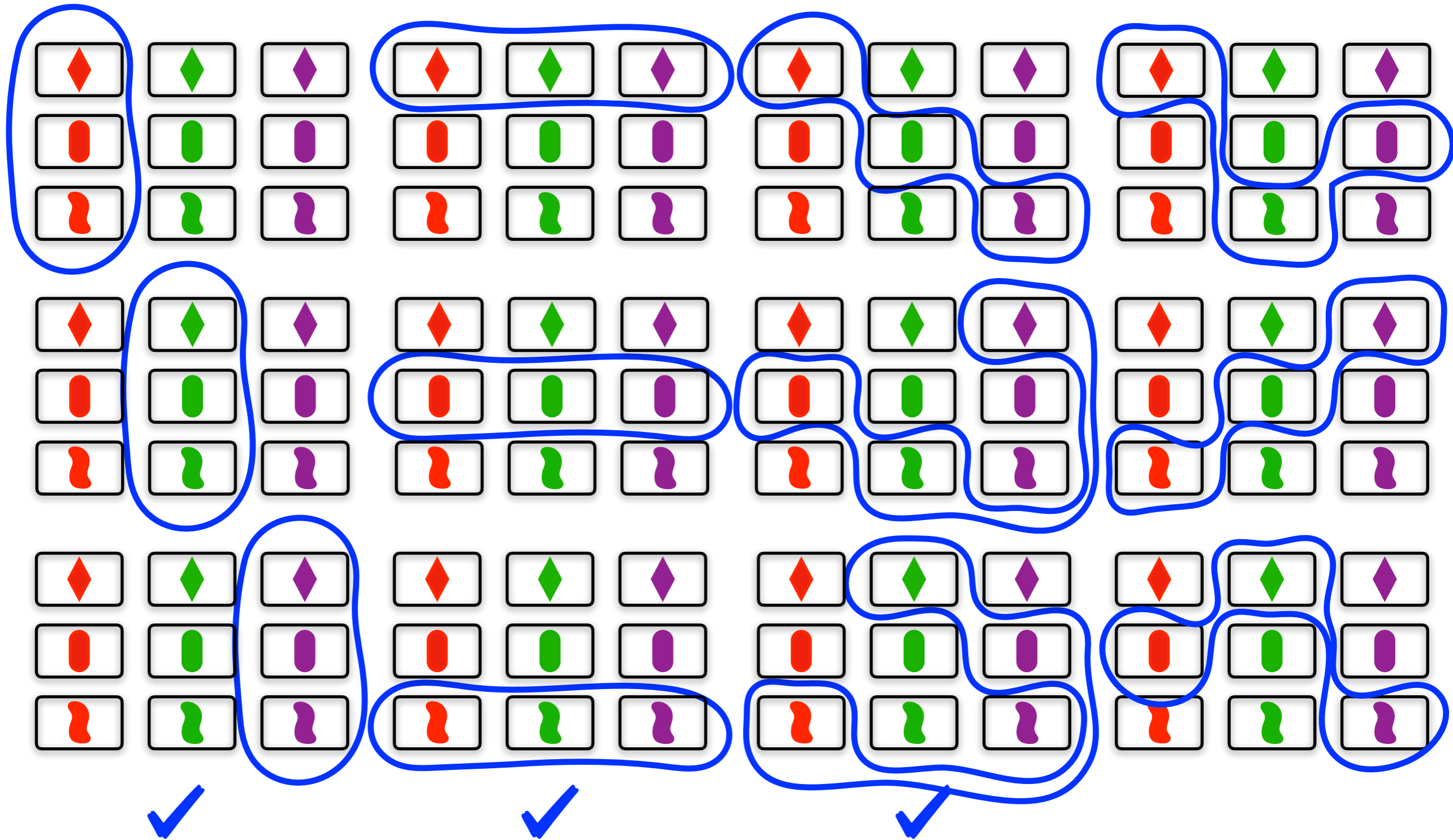




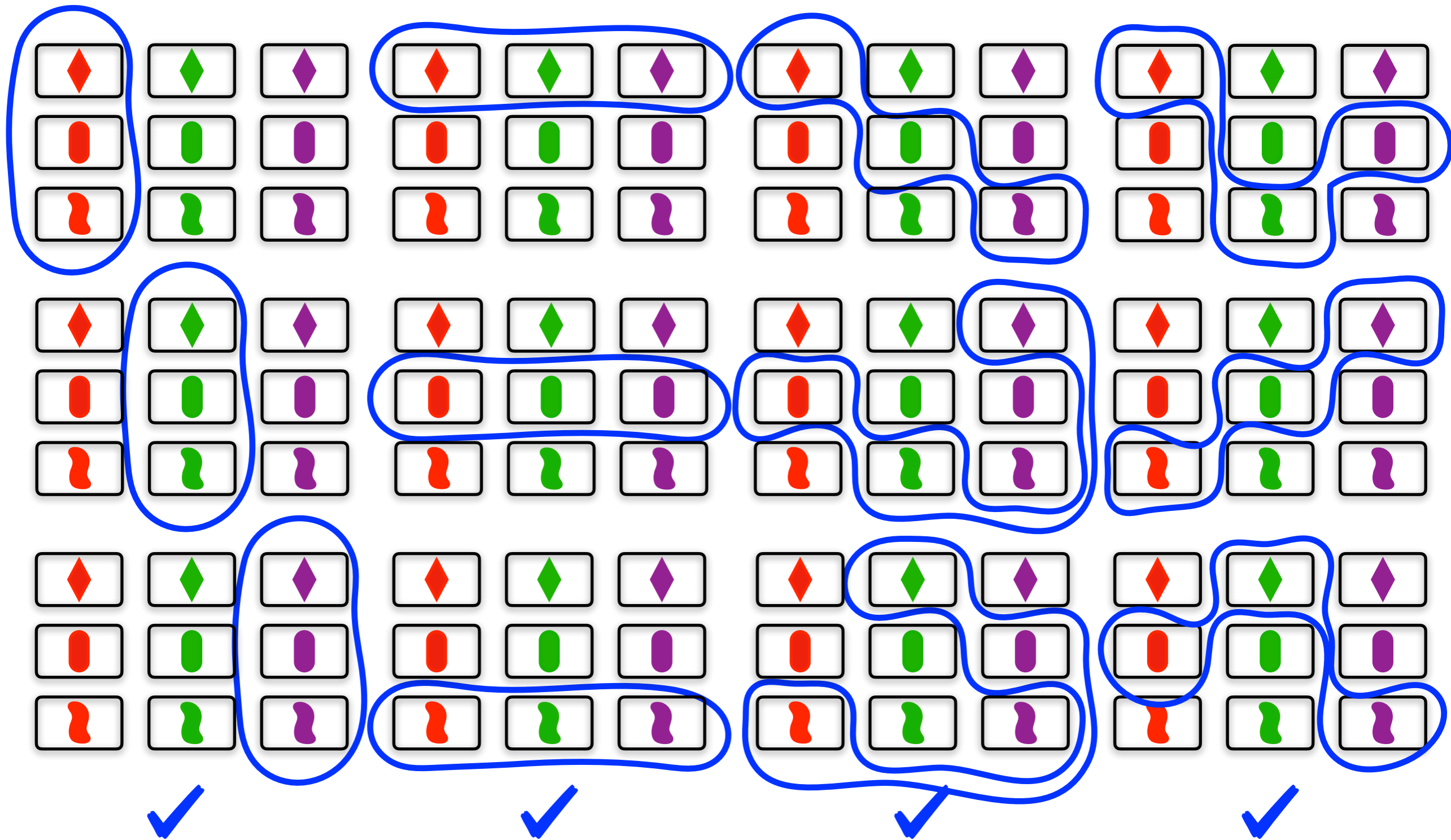




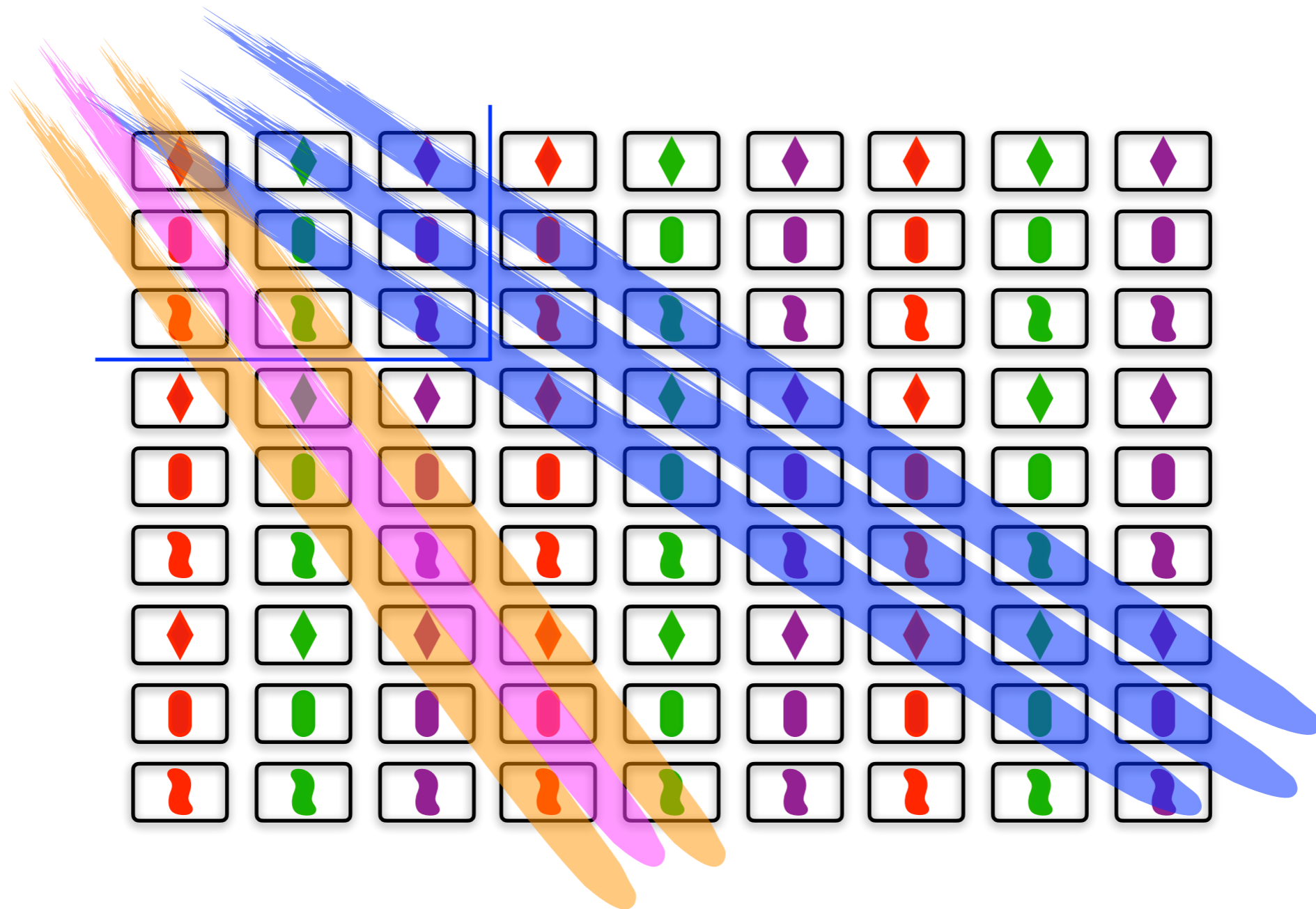
# What are the possible SETs?



# What are the possible SETs?



We observed that SETs lie on lines!



**Is this a real thing?!?**

**Is this a real thing?!?**

Why can we just repeat the grid like that?

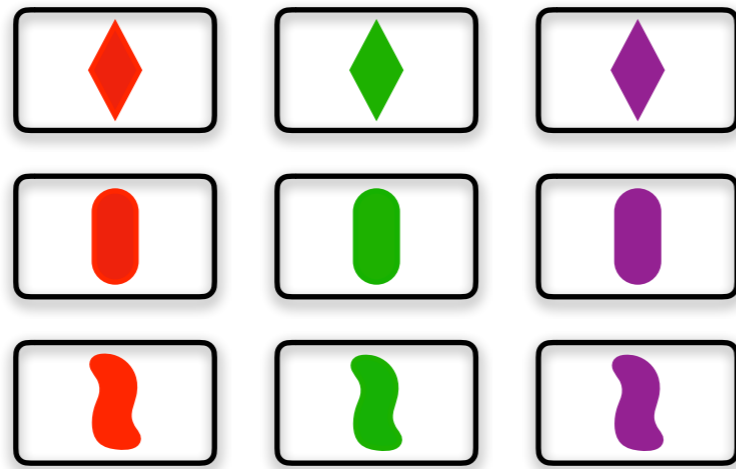


**Is this a real thing?!?**

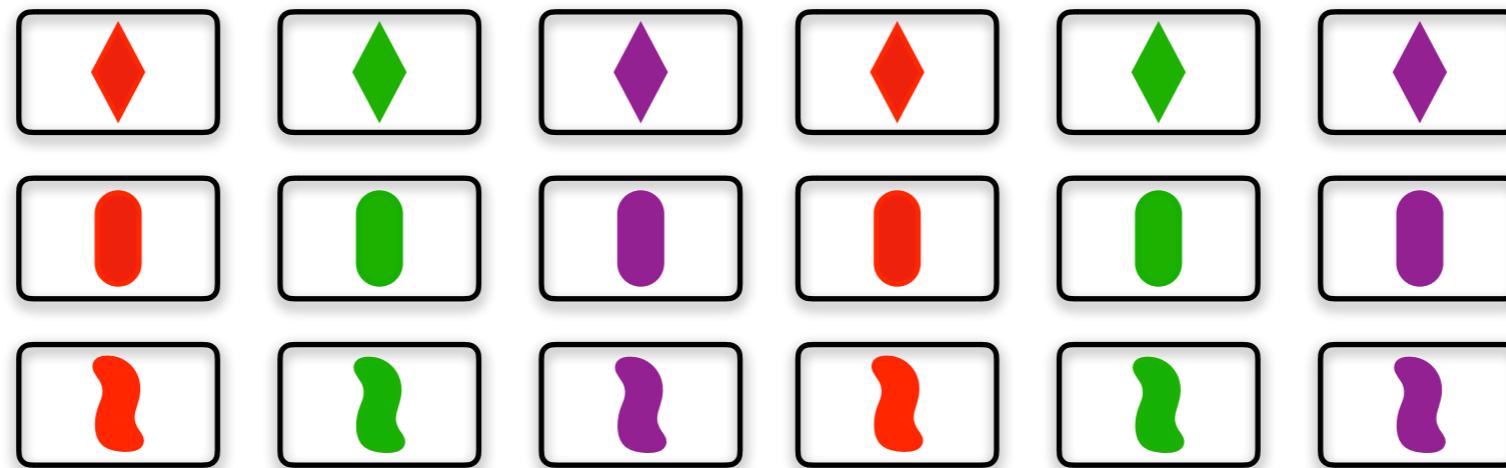
Why can we just repeat the grid like that?

Does every SET have to lie on a line? Or is it just the ones that we found?

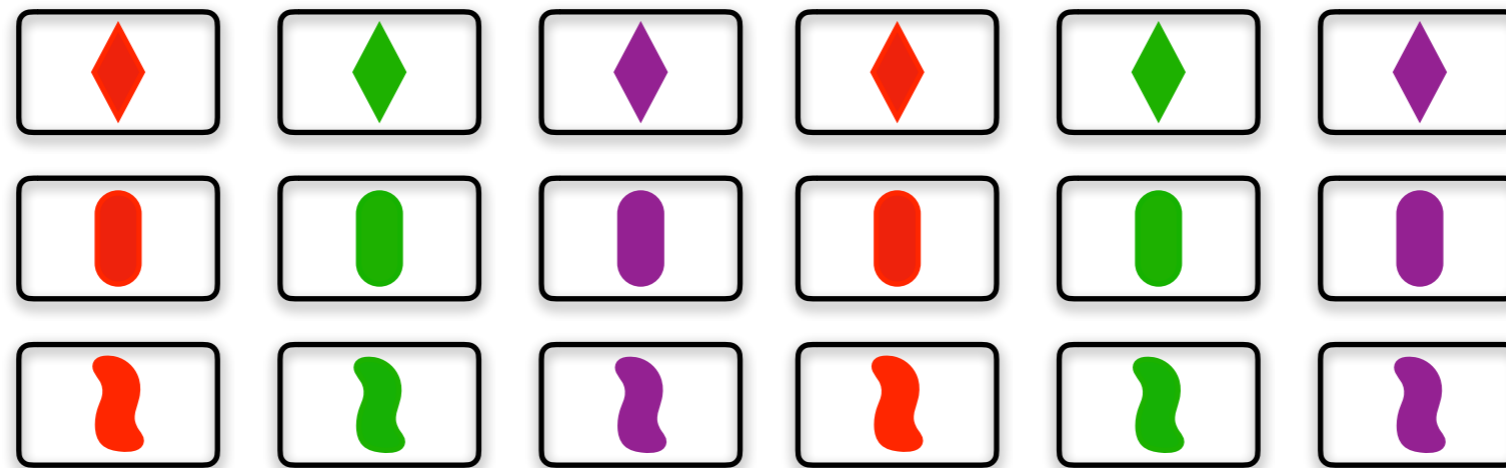
Why can we just repeat the grid like that?



Why can we just repeat the grid like that?



Why can we just repeat the grid like that?



Why can we just repeat the grid like that?



14 is the same as 2

26 is the same as 2





14 is the same as 2

26 is the same as 2



14 is the same as 2

26 is the same as 2

This is called working ***mod 12***.

We do usual arithmetic, but treat 12 as 0.



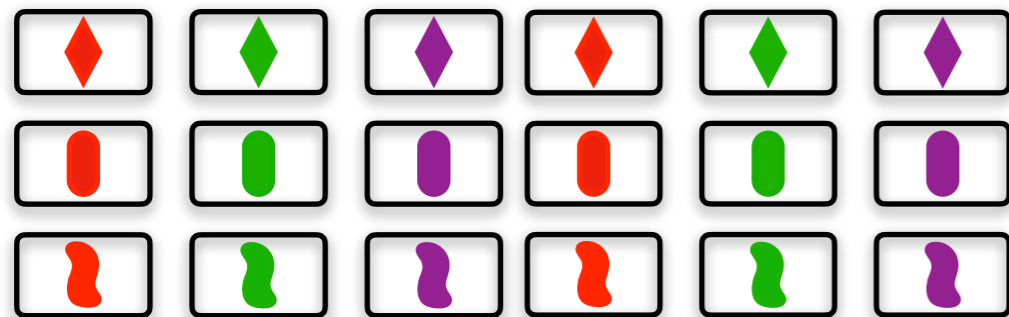


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This is called working *mod 12*.

We do usual arithmetic, but treat 12 as 0.



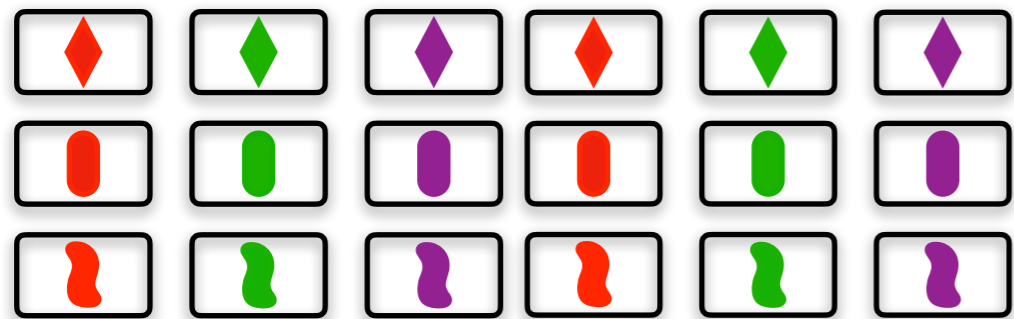


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This is called working *mod 12*.

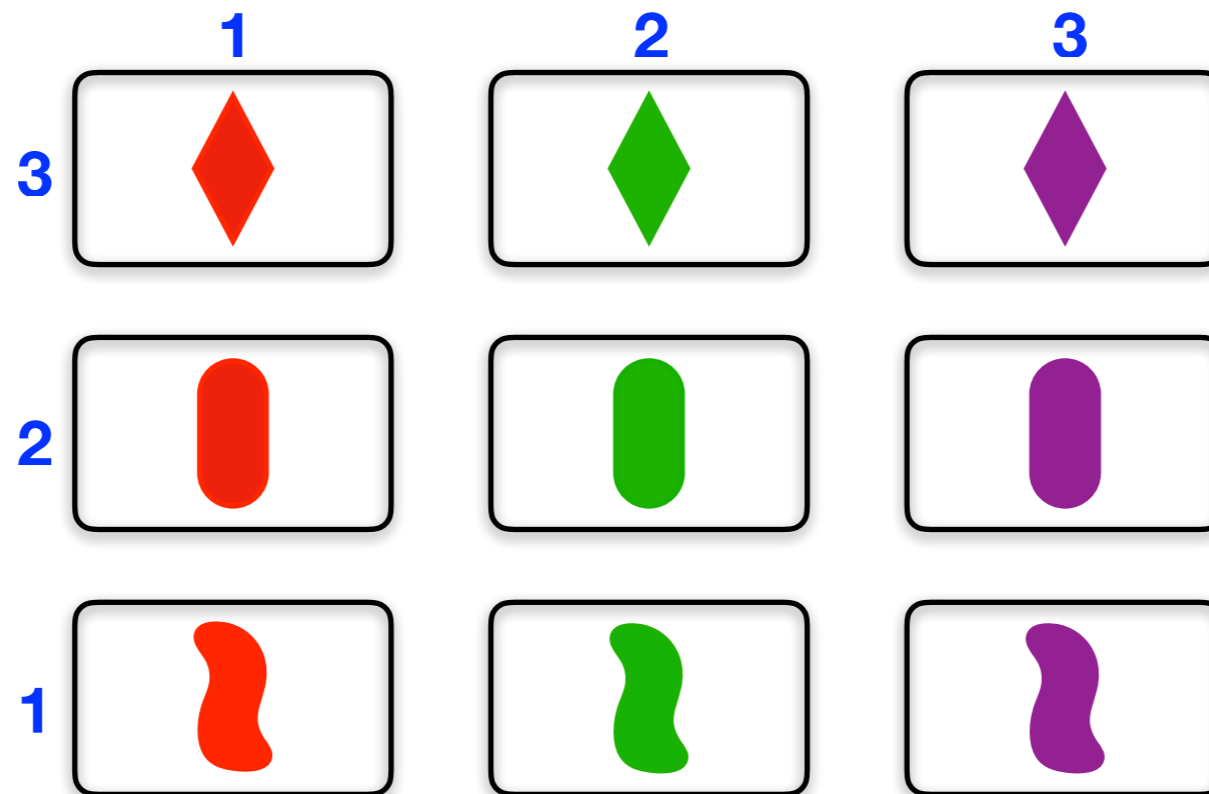
We do usual arithmetic, but treat 12 as 0.



It makes sense to repeat  
when we work *mod 3*!

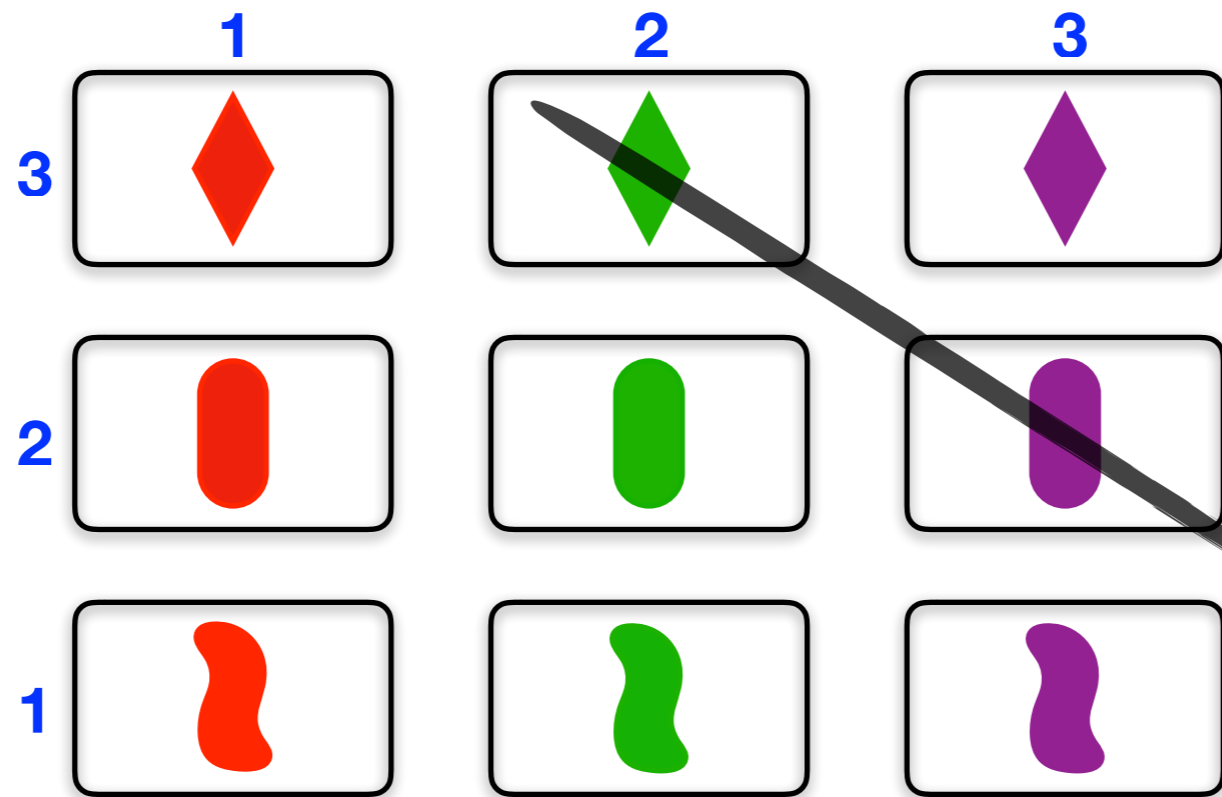
# Is this a real thing?!?

Why can we just repeat the grid like that?



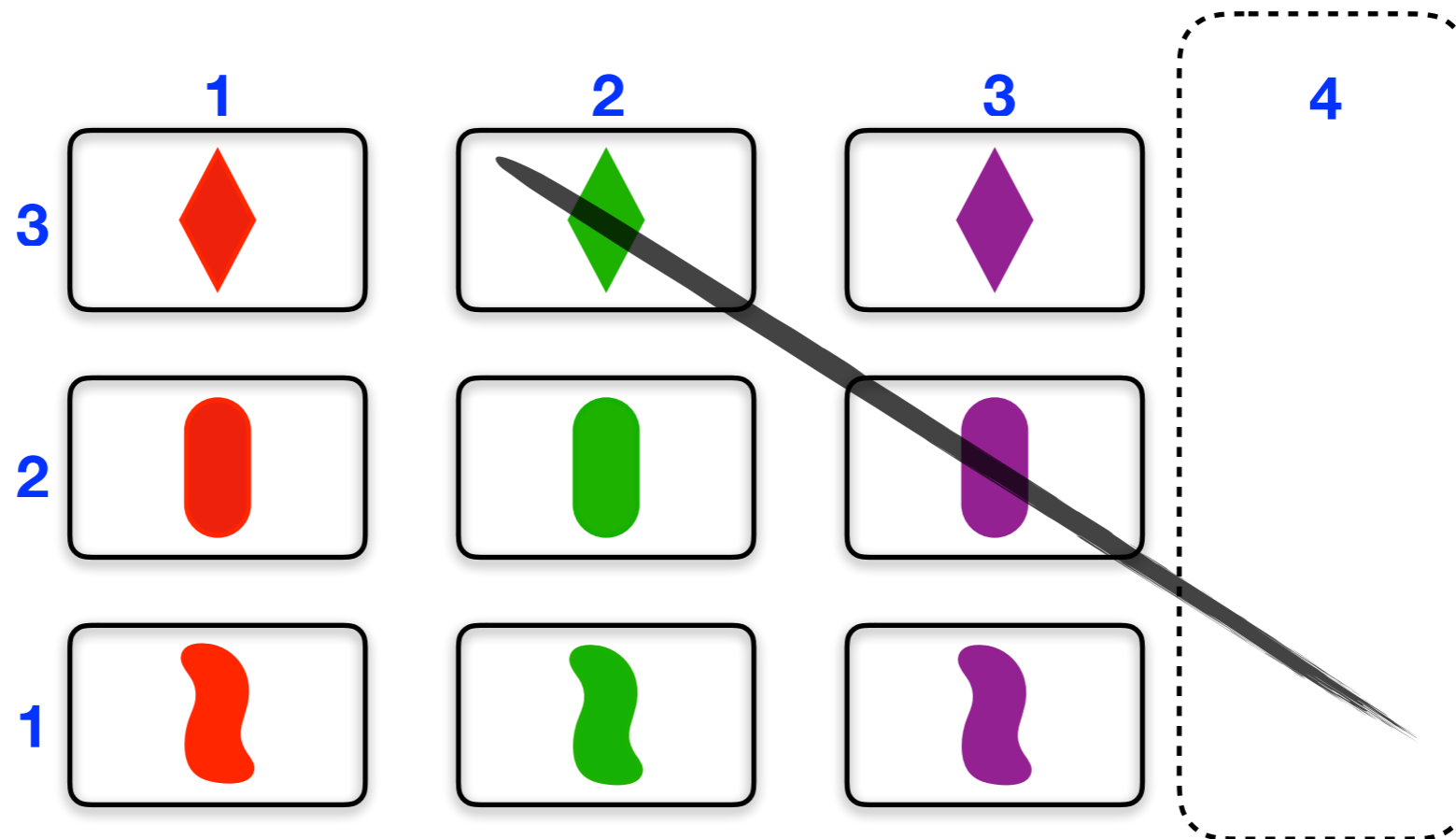
# Is this a real thing?!?

Why can we just repeat the grid like that?



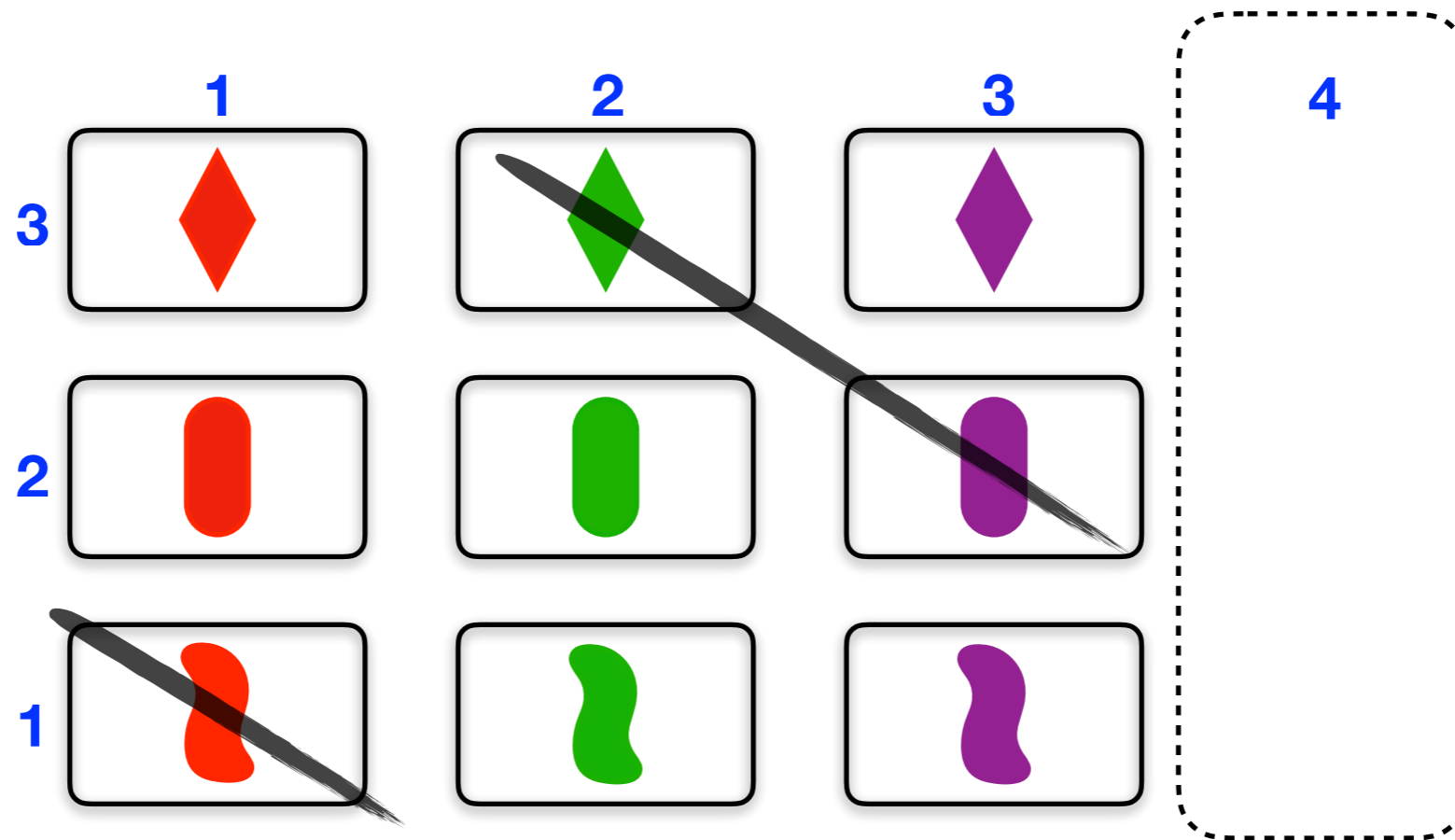
# Is this a real thing?!?

Why can we just repeat the grid like that?



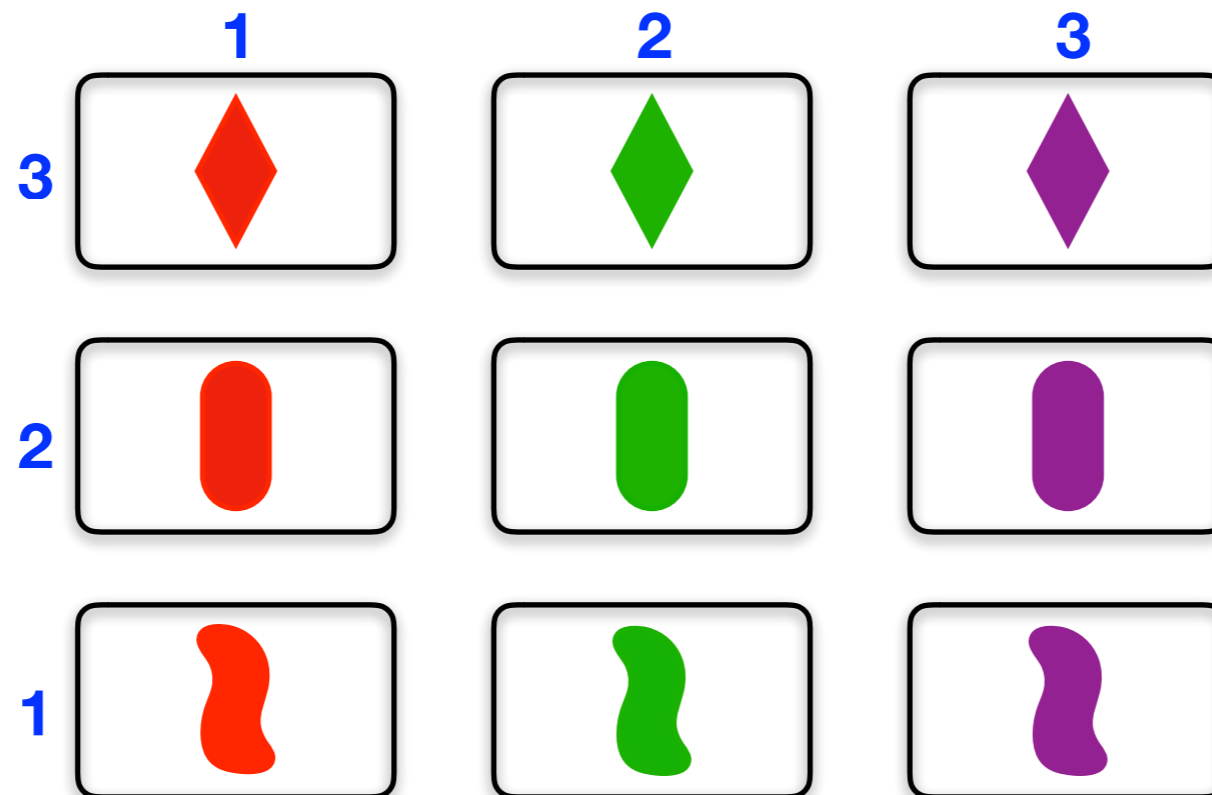
# Is this a real thing?!?

Why can we just repeat the grid like that?



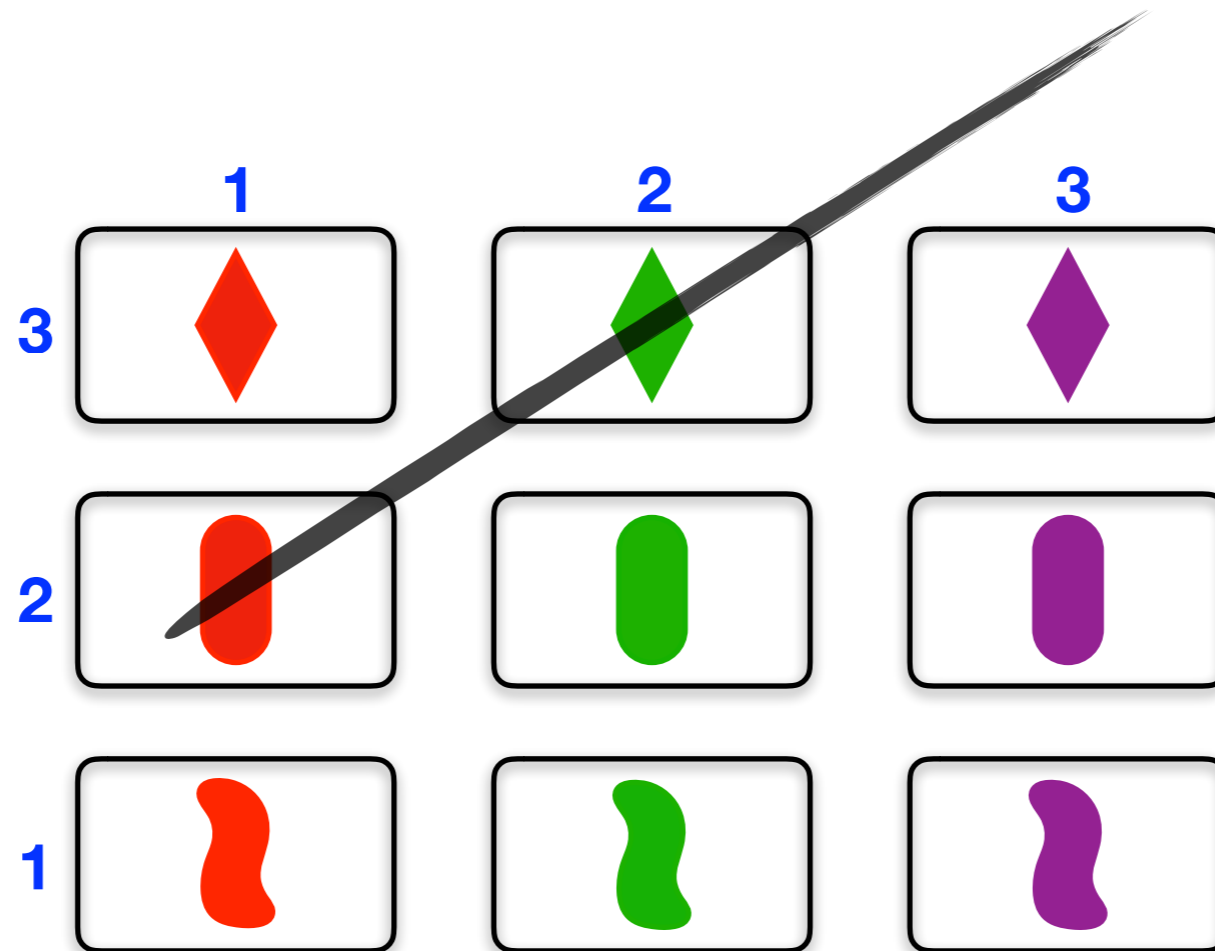
# Is this a real thing?!?

Why can we just repeat the grid like that?



# Is this a real thing?!?

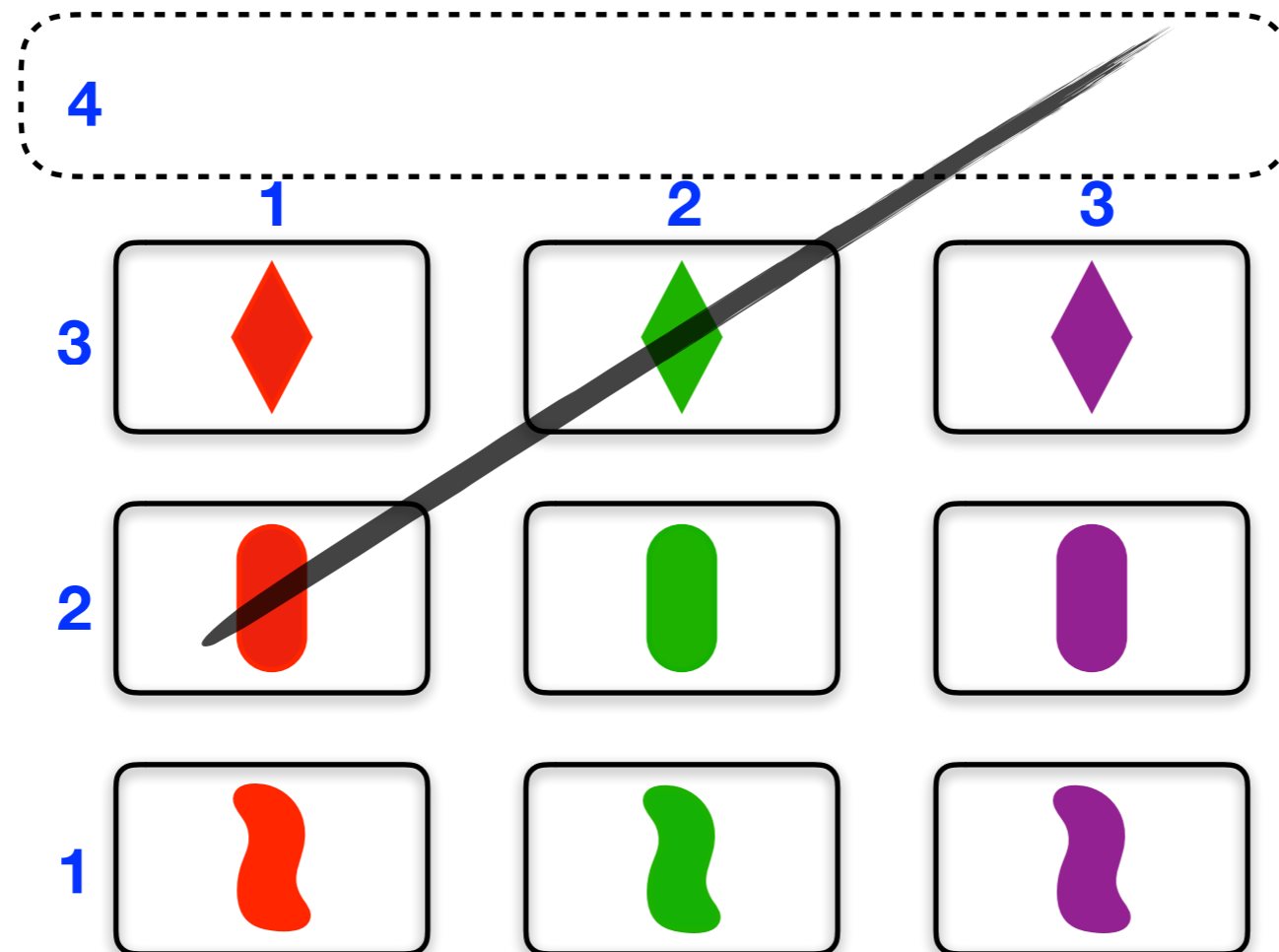
Why can we just repeat the grid like that?





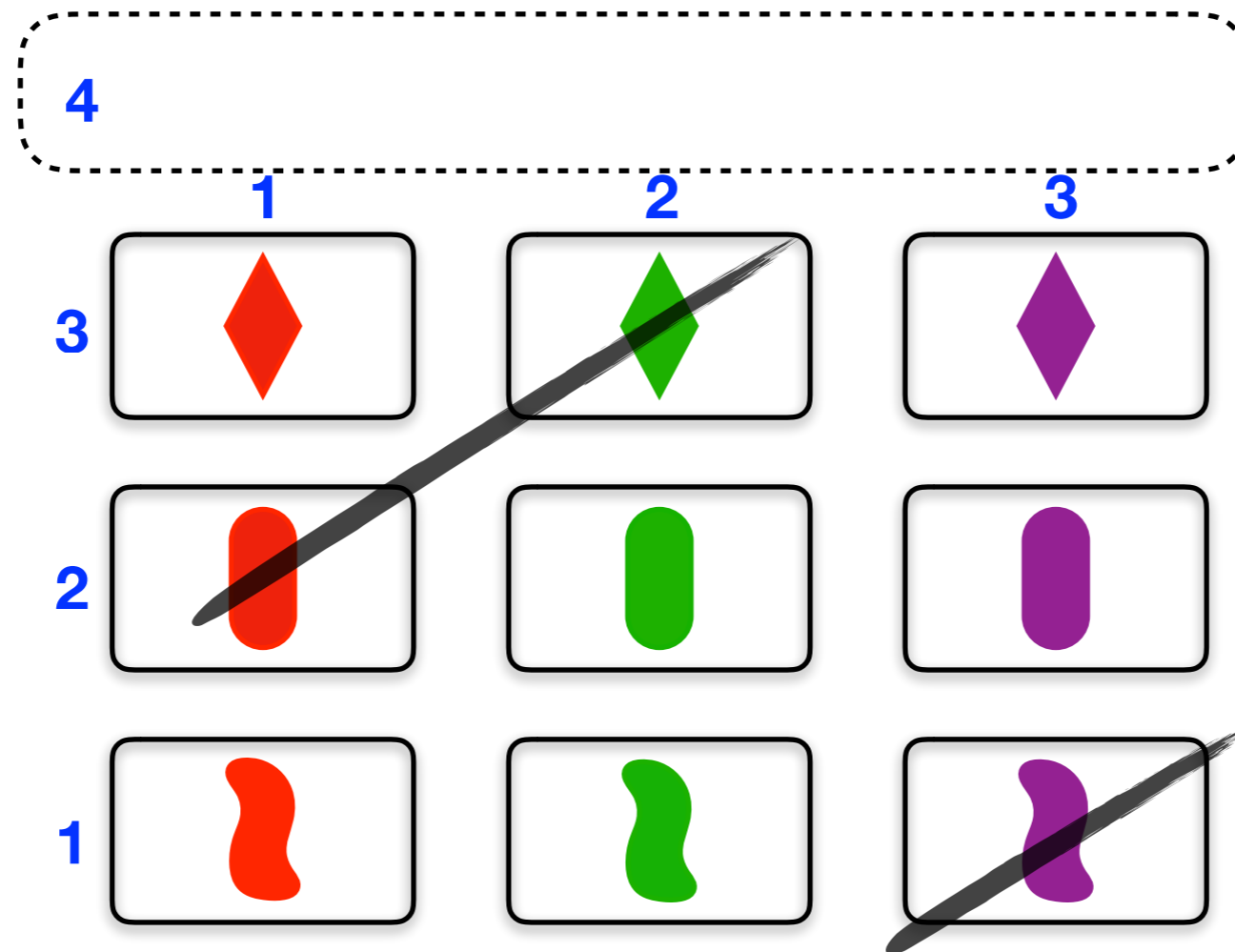
# Is this a real thing?!?

Why can we just repeat the grid like that?



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Does every SET have to lie on a line? Or is it just the ones that we found?

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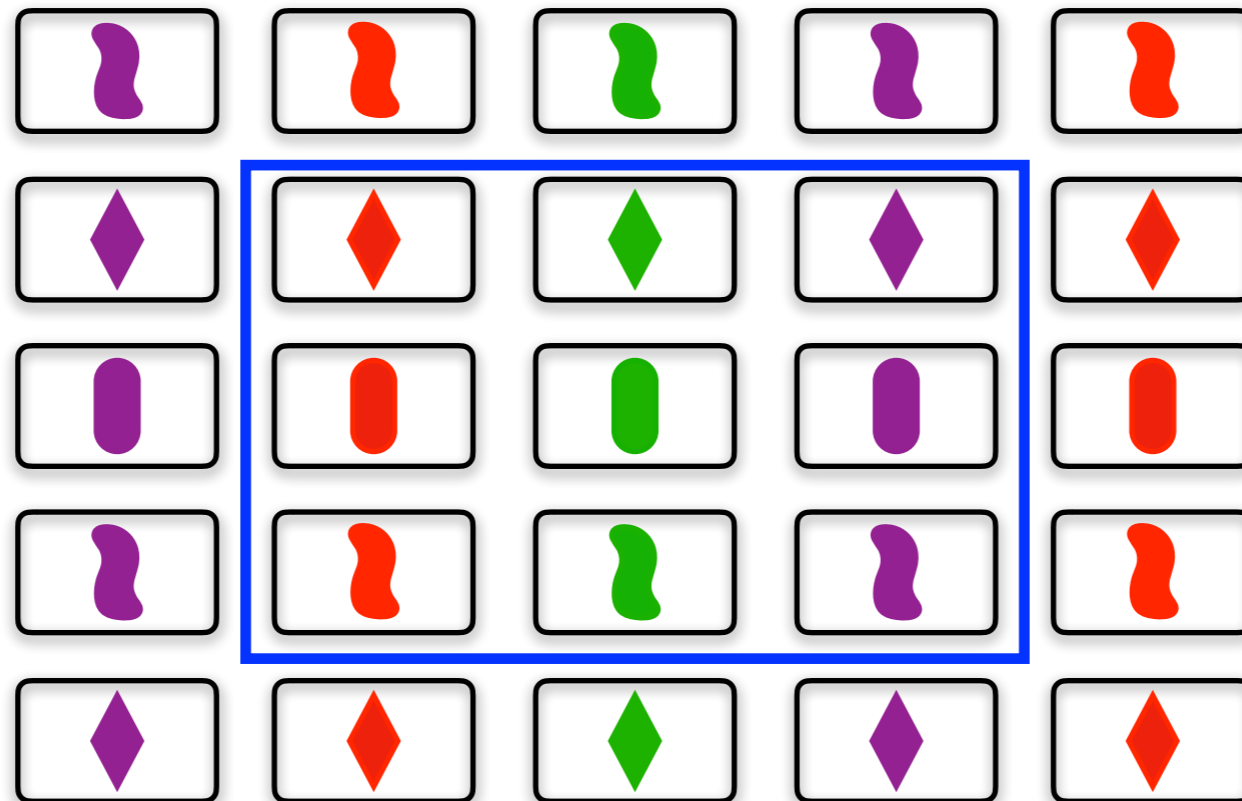
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The only possible slopes are 1, 0, -1, or  $\infty$

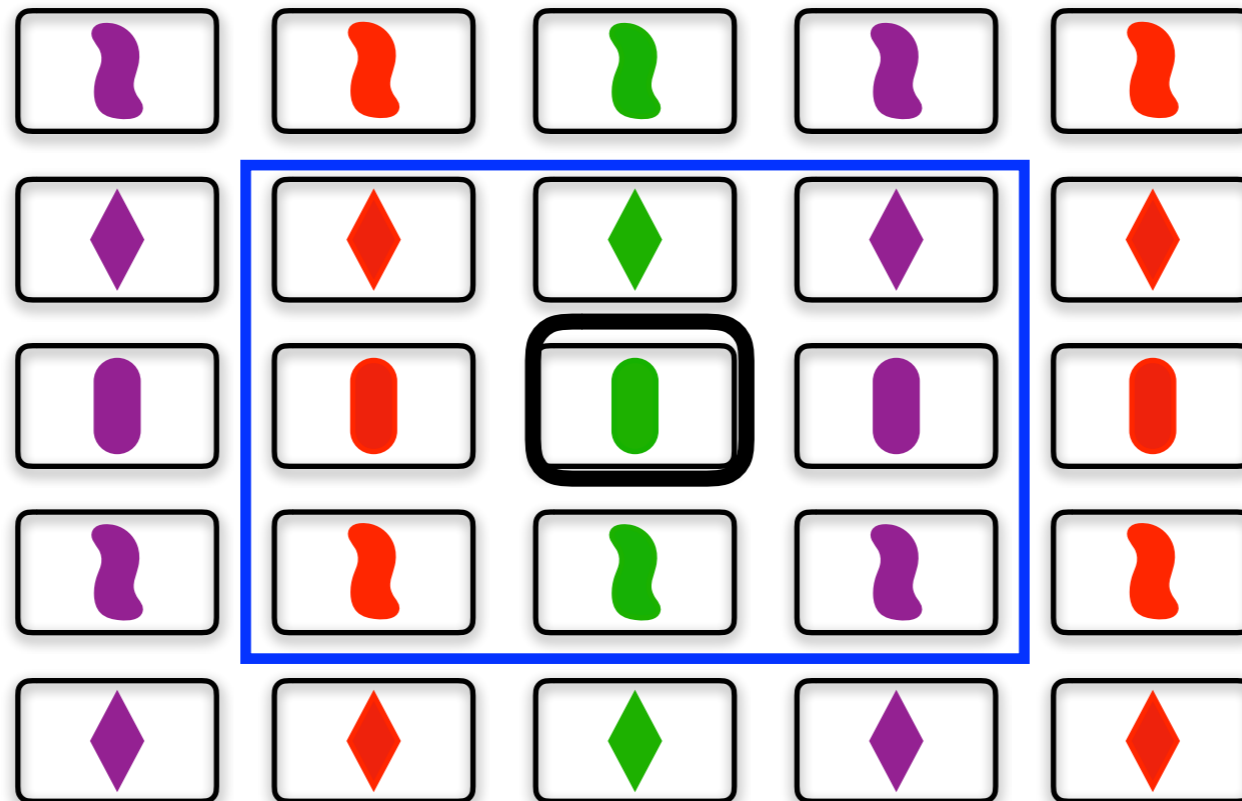
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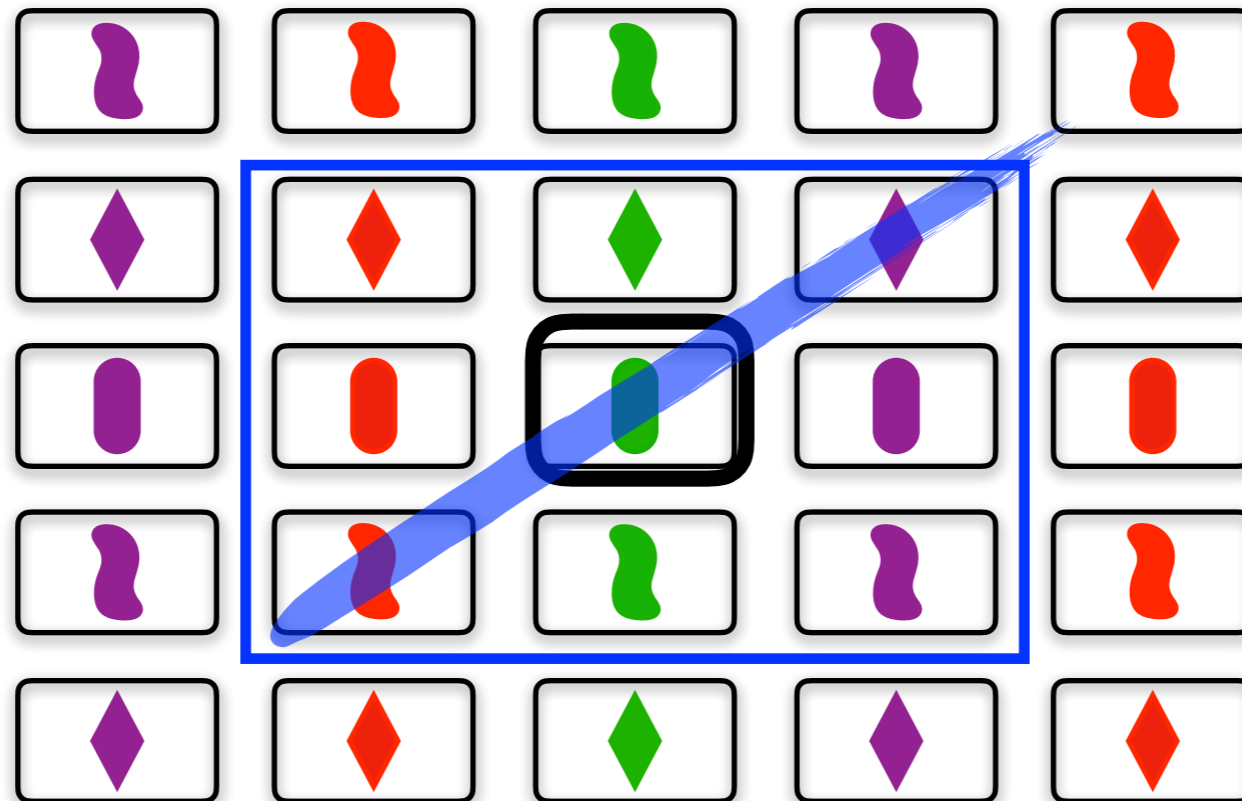
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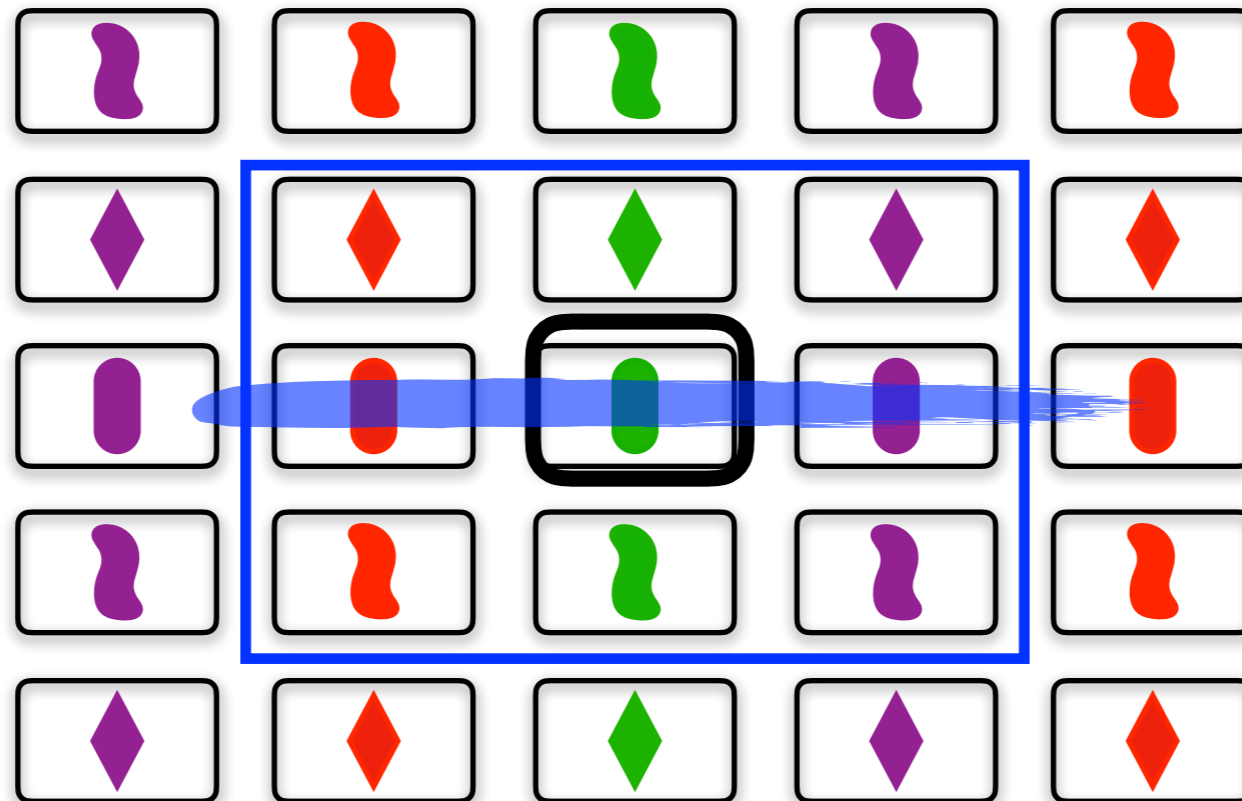
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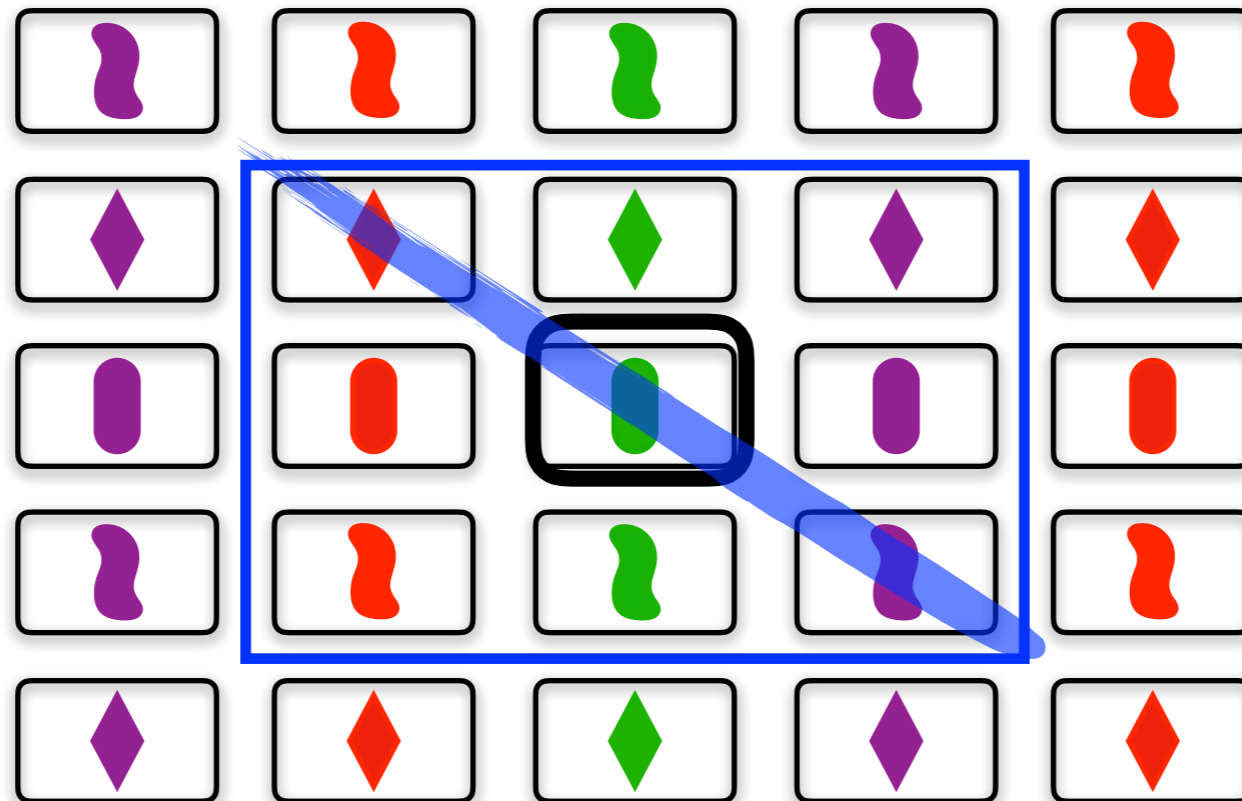
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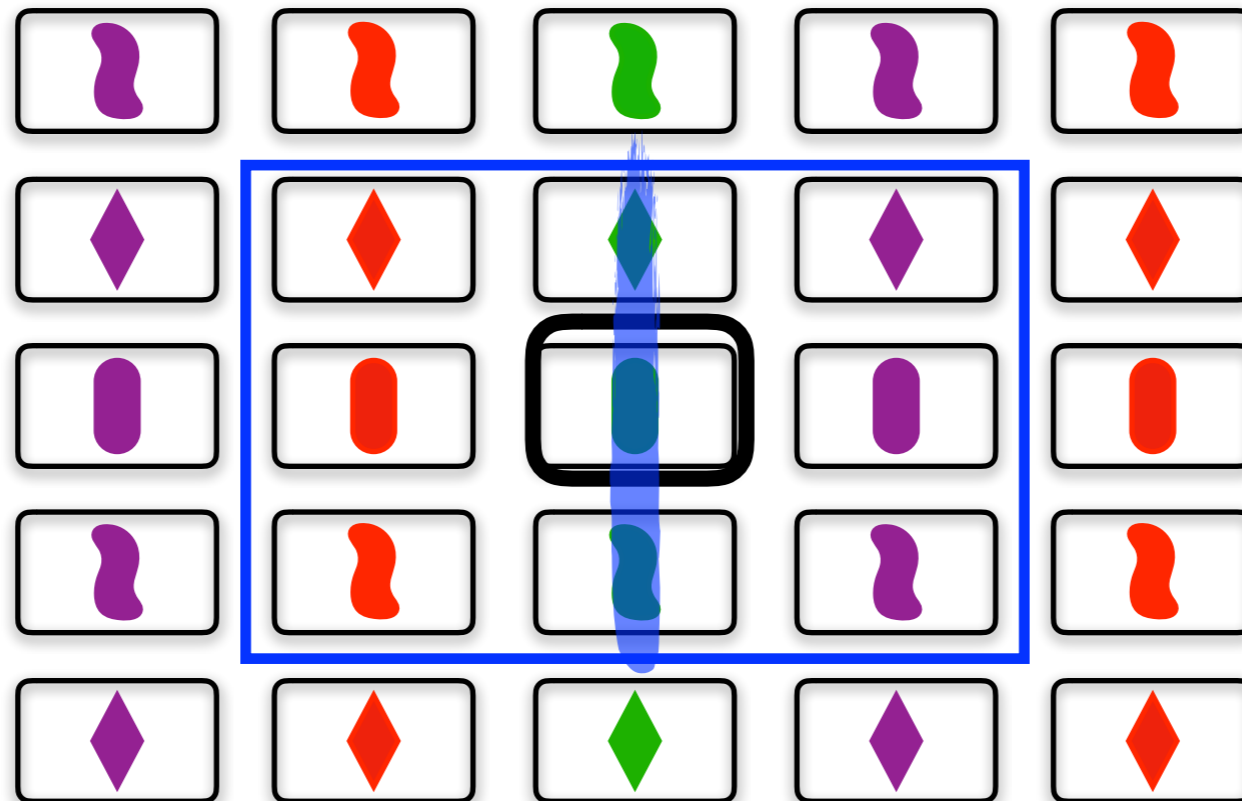
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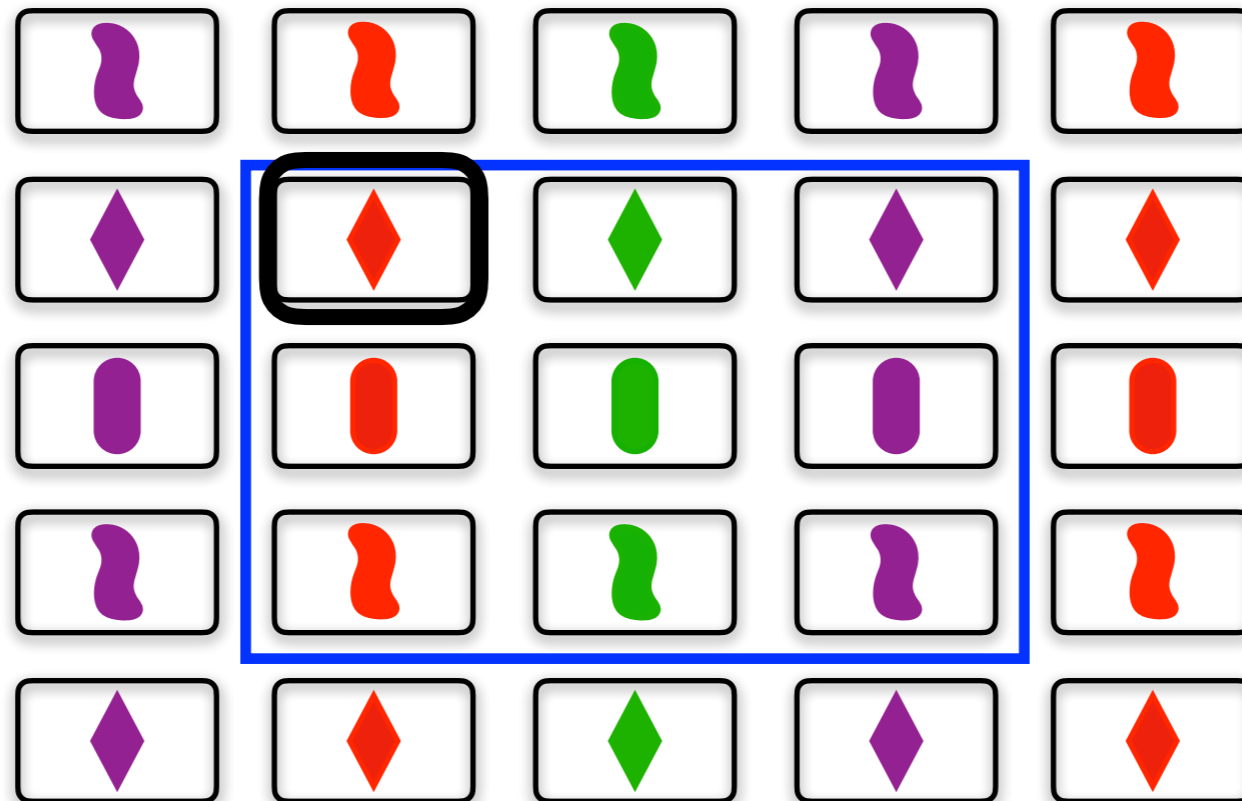
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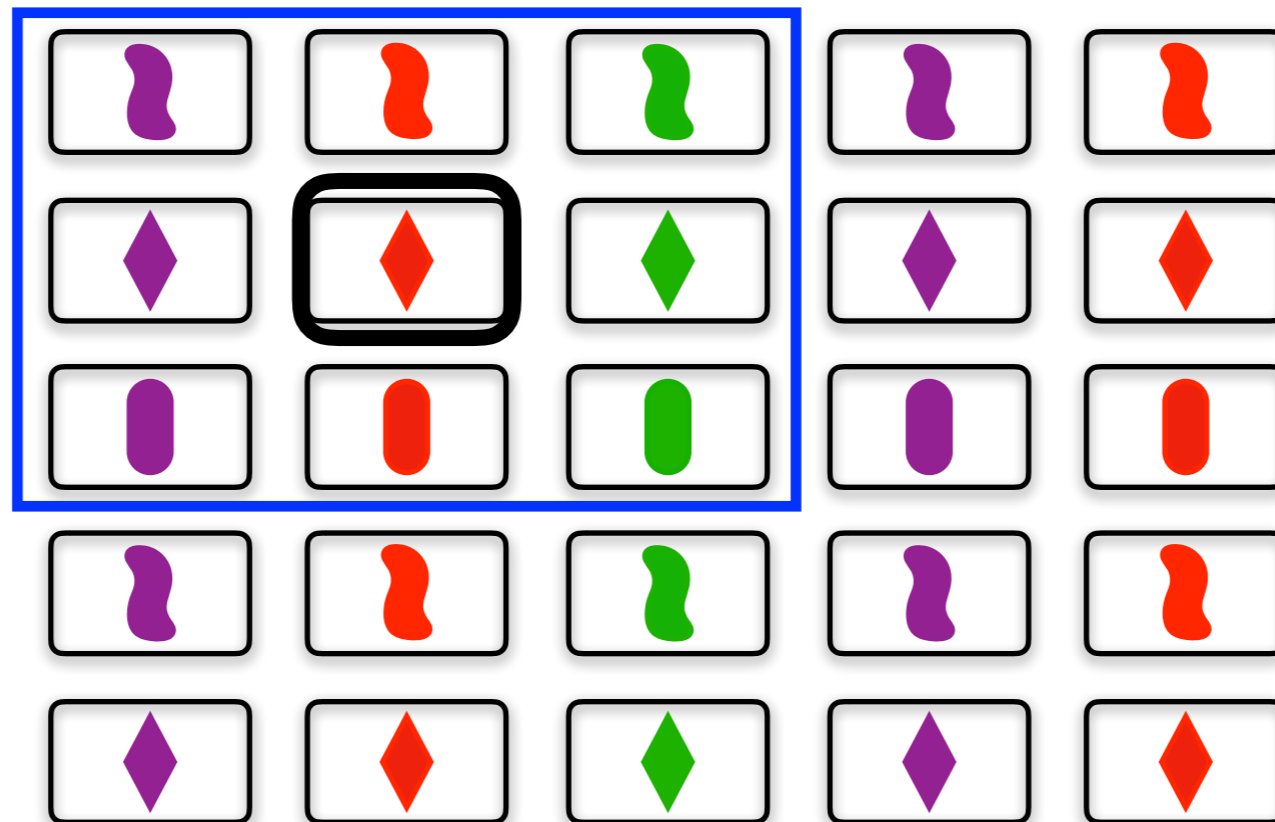
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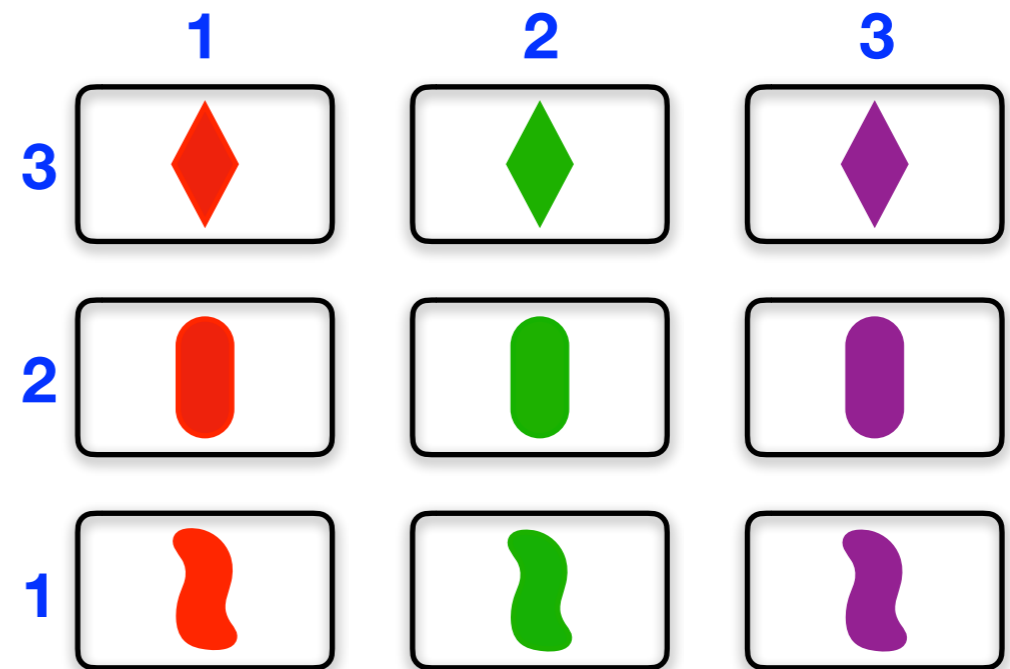
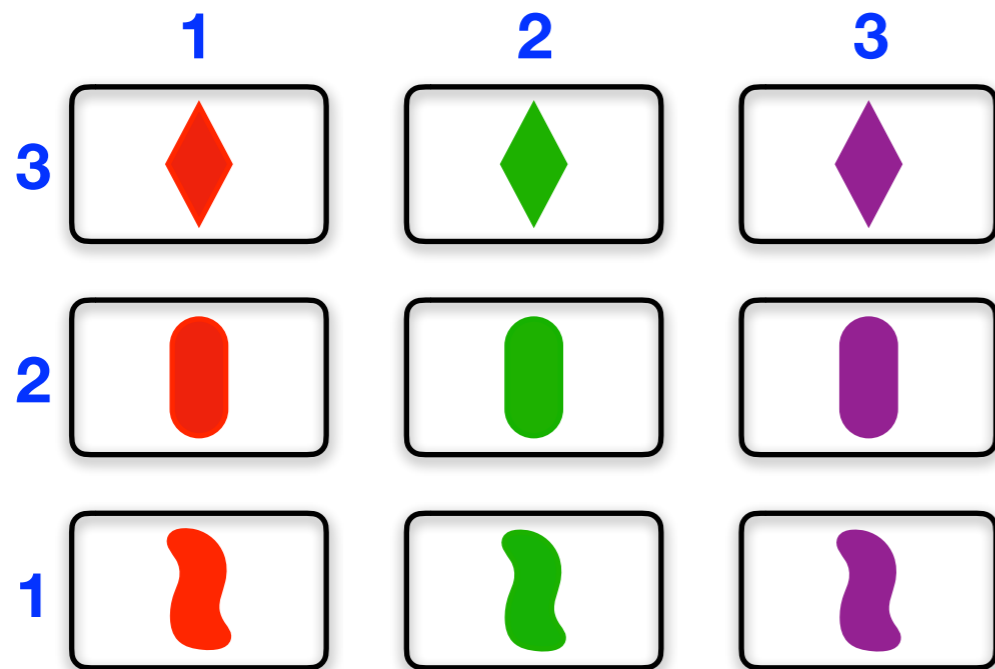
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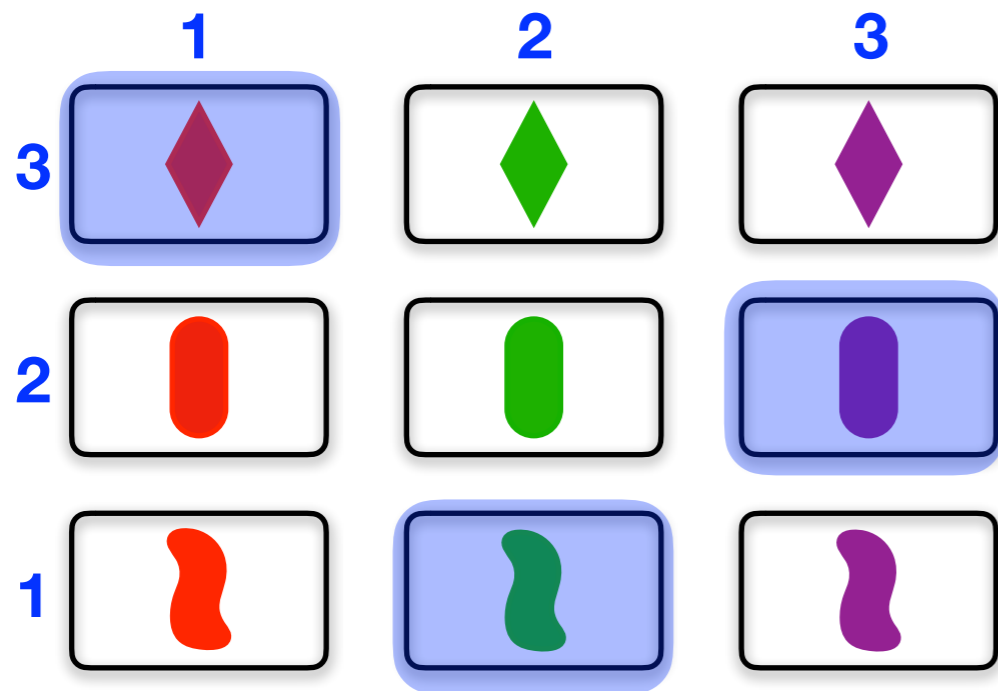
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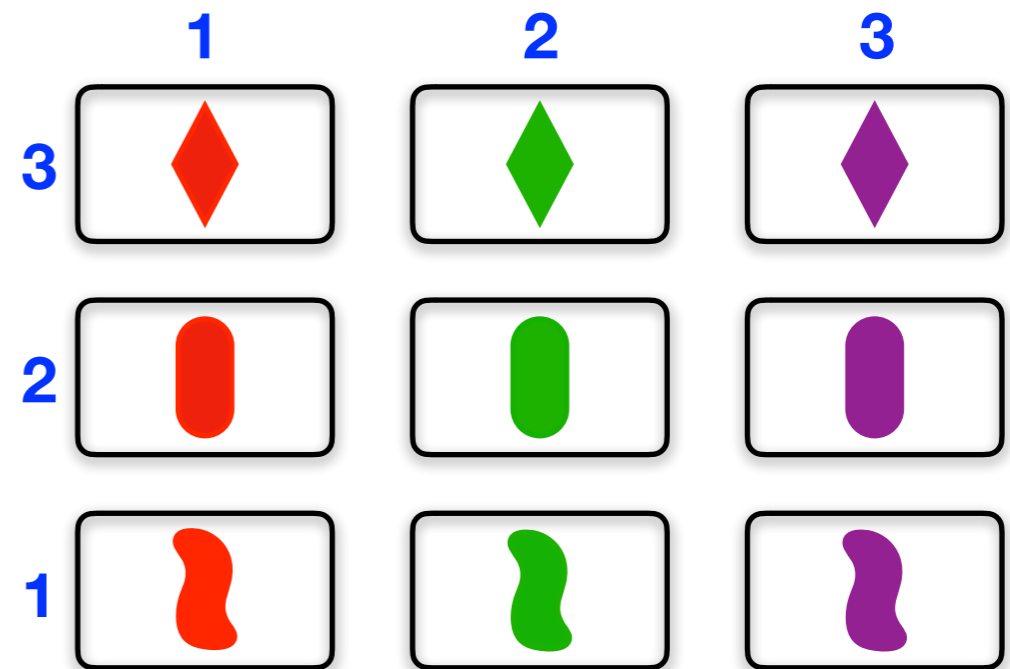




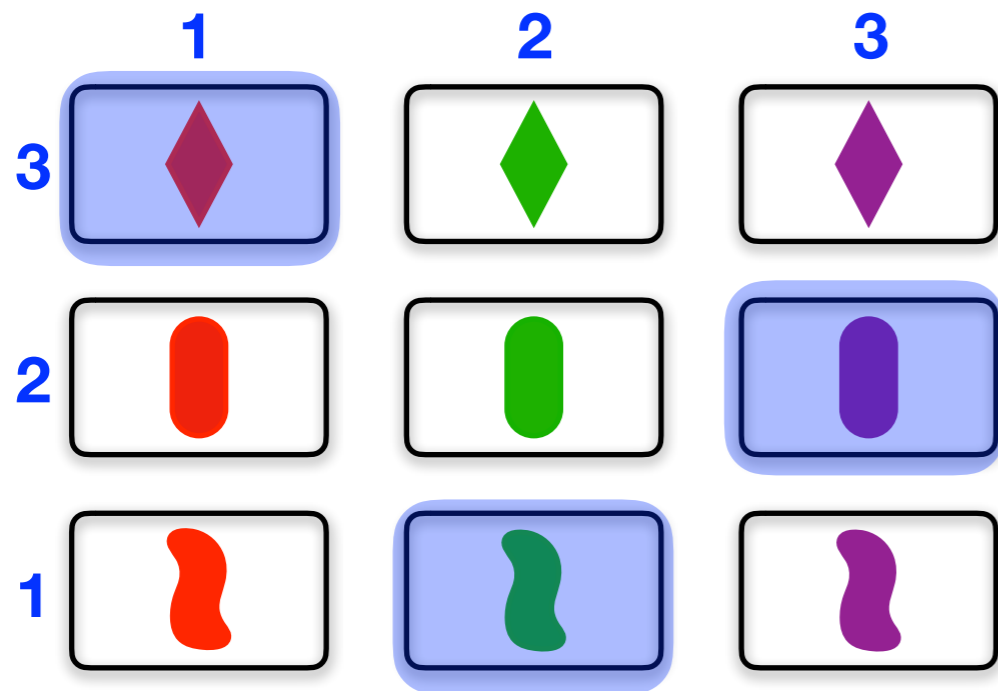
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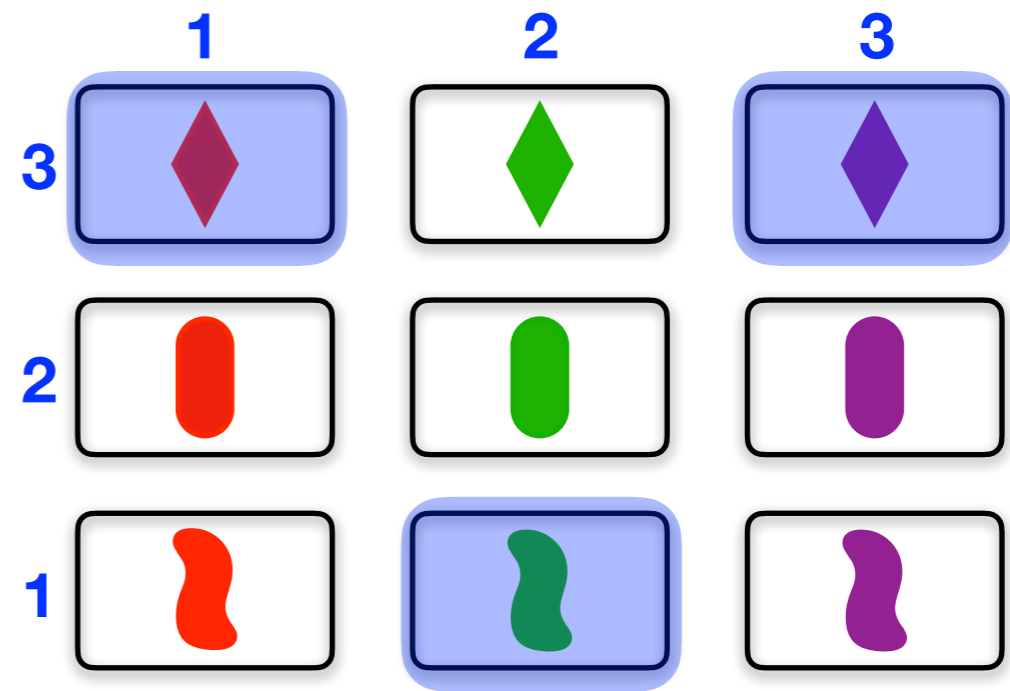
**SET**



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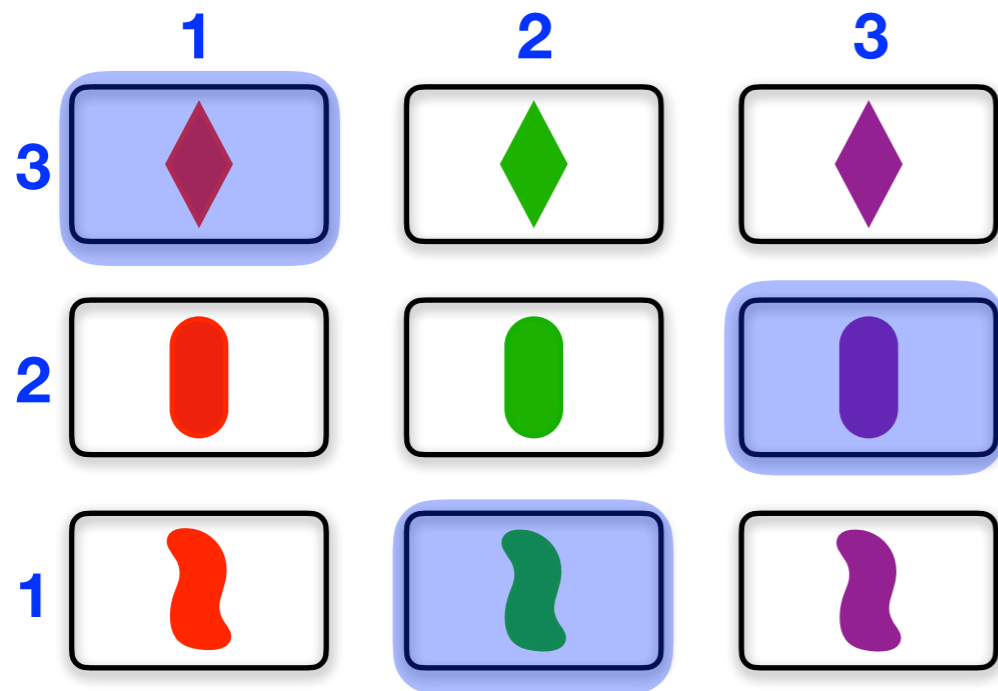


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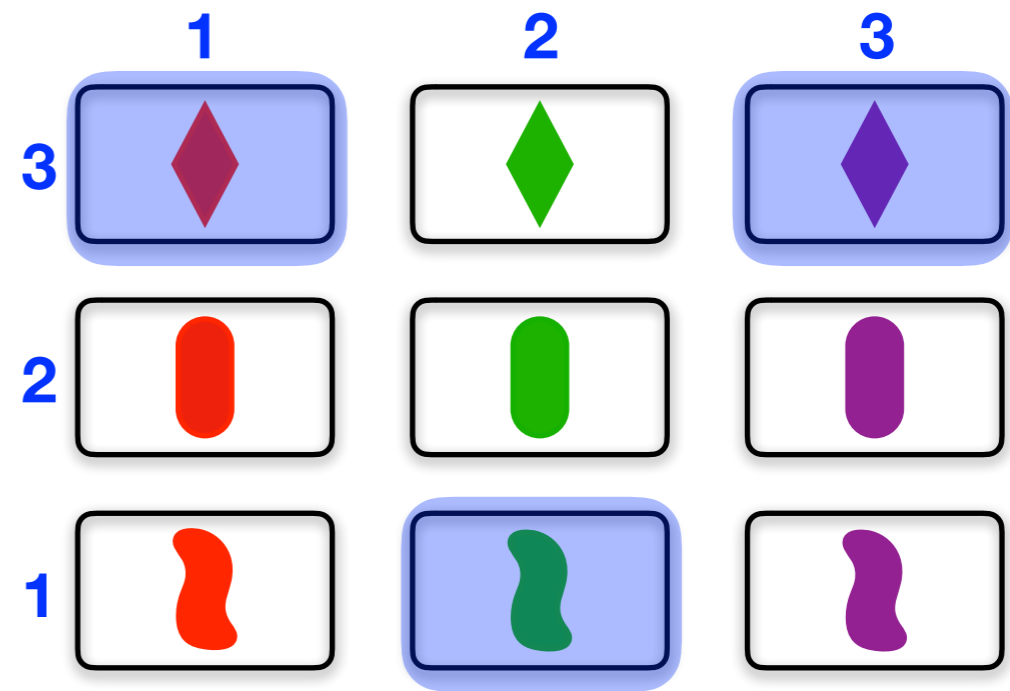
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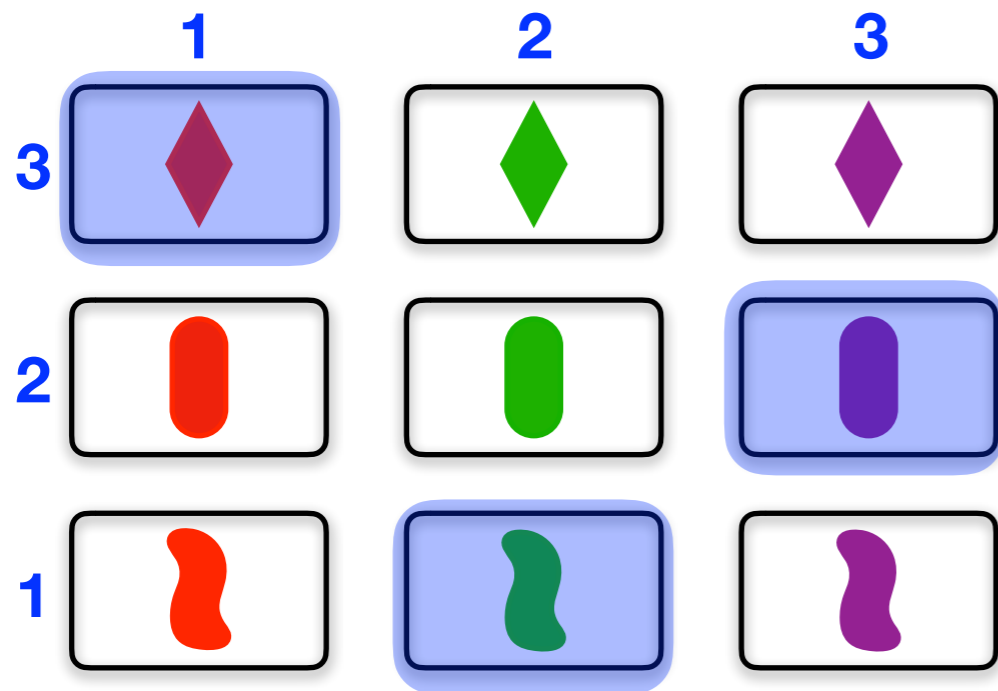
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slopes are 1, 1, 1



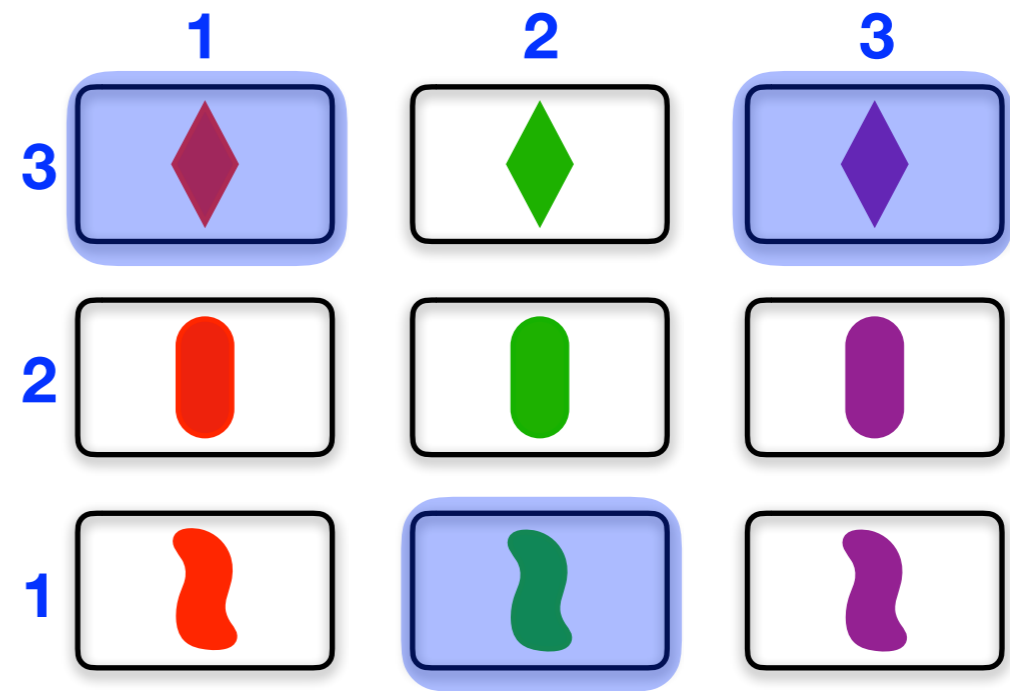
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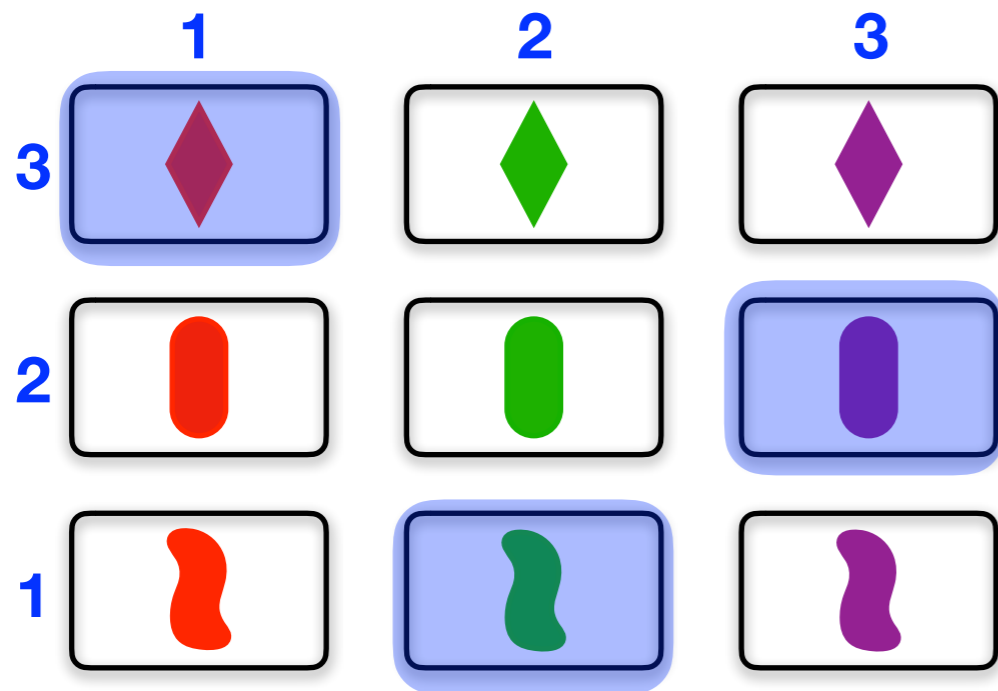
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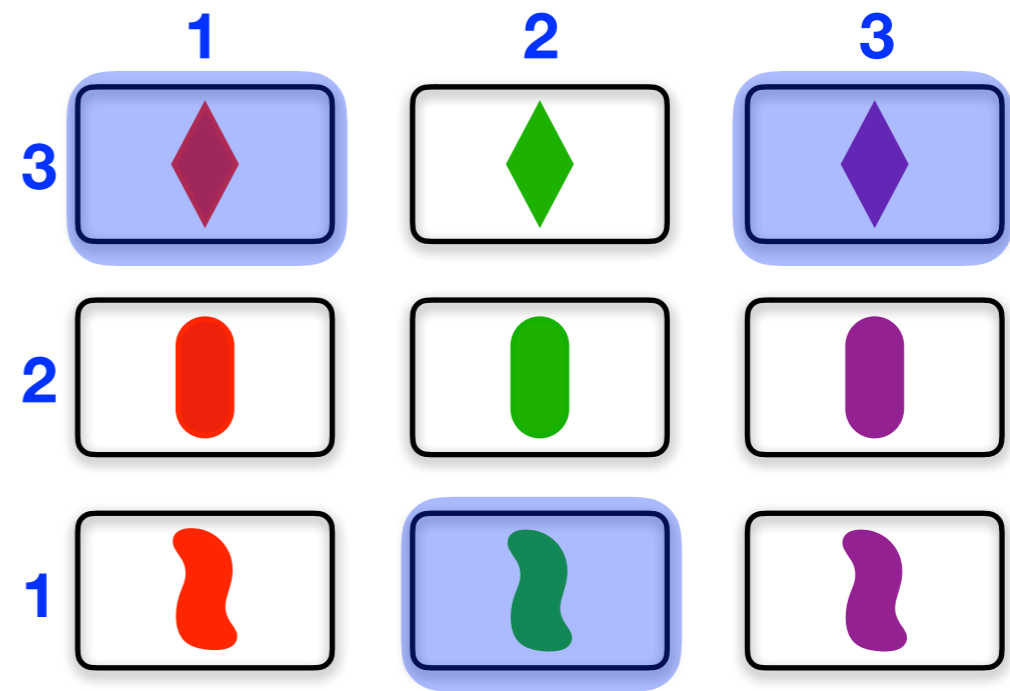
slopes are 0, 1, -1

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**SET**

slopes are 1, 1, 1



**NOT A SET**

slopes are 0, 1, -1

If 3 points lie on a line, then all pairs have the same slope

If 3 points do not lie on a line, then all pairs have different slopes

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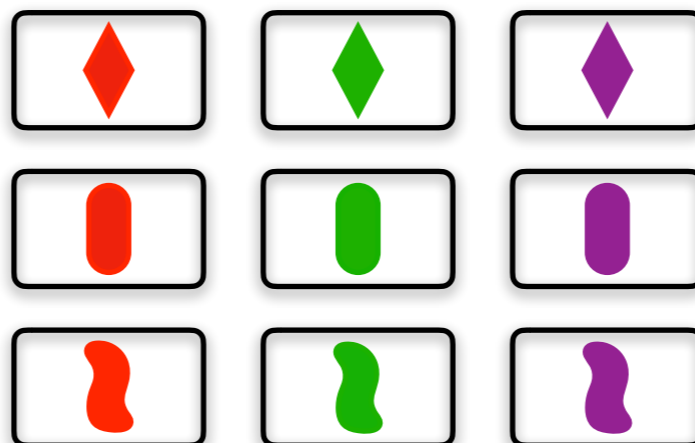
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