## Finding patterns in differences

The mathematics behind the game SET

## What is SET?

## What is SET?

## Is there a card that doesn't belong?

## What is SET?

## Is there a card that doesn't belong?



## What is SET?

## Is there a card that doesn't belong?



Not a SET!

## What is SET?

## Is there a card that doesn't belong? <br>  <br> Not a SET! <br> 

## What is SET?

## Is there a card that doesn't belong?



## What is SET?

## Is there a card that doesn't belong?


$\uparrow$

## What is SET?

## Is there a card that doesn't belong?



NOT A SET!


## What is SET?

## Is there a card that doesn't belong?



## What is SET?

## Is there a card that doesn't belong?



## What is SET?

## Is there a card that doesn't belong?



## What is SET?

## Is there a card that doesn't belong?



## What is SET?

## Is there a card that doesn't belong?


$\checkmark$

$\checkmark$
$\checkmark$

## What is SET?

## Is there a card that doesn't belong?


$\checkmark$

$\checkmark$


A SET!!

## What is SET?

## Is there a card that doesn't belong?



## What is SET?

## Is there a card that doesn't belong?


$\checkmark$

$\checkmark$

## What is SET?

## Is there a card that doesn't belong?


$\checkmark$

$\checkmark$
A SET!!

## What is SET?

## Is there a card that doesn't belong?



## What is SET?

## Is there a card that doesn't belong?


$\checkmark$


## What is SET?

## Is there a card that doesn't belong?


$\checkmark$


A SET!!

## Each card in SET has 4 properties

Color


Number


## Each card in SET has 4 properties

Color


Number


Have $3 \times 3 \times 3 \times 3=3^{4}=81$ cards
No repeats!
"No card that doesn't belong"
$\downarrow$
"For every property, all are the same or all are different"

Color


## For every property, all are the same or all are different



## For every property, all are the same or all are different

Two are red, one is green


## For every property, all are the same or all are different

Two are red, one is green


NOT A SET!

## For every property, all are the same or all are different



## For every property, all are the same or all are different

Two are solid, one is empty


## For every property, all are the same or all are different

Two are solid, one is empty


NOT A SET!

## For every property, all are the same or all are different



## For every property, all are the same or all are different

All solid, all green, all diamonds, all different numbers


## For every property, all are the same or all are different

All solid, all green, all diamonds, all different numbers


A SET!!

## For every property, all are the same or all are different



## For every property, all are the same or all are different

All the same number, all solid, all different shapes, all different colors


## For every property, all are the same or all are different

## All the same number, all solid, all different shapes, all different colors



A SET!!

## For every property, all are the same or all are different



## For every property, all are the same or all are different

Same number, all different shading, all different shapes, all different colors


## For every property, all are the same or all are different

Same number, all different shading, all different shapes, all different colors


A SET!!

## For every property, all are the same or all are different



## For every property, all are the same or all are different

All different numbers, all different colors, all different shapes, all different shading


## For every property, all are the same or all are different

All different numbers, all different colors, all different shapes, all different shading


A SET!!

## How many SETs are there?



## What are the possible SETs?



## What are the possible SETs?



## What are the possible SETs?





$$
\begin{aligned}
& \square \square-\square \cdot \square \square \square \square \square \\
& 00 \square 010 \\
& \square \\
& \square \\
& \square \\
& \square
\end{aligned}
$$

$$
\begin{aligned}
& \begin{array}{|l|l|}
\hline 8,8 \\
\hline-7
\end{array} \\
& 8 \\
& \square \\
& \square \\
& \square \\
& \square \\
& \square \\
& \square \\
& \square \\
& \square \\
& \square \\
& \square \\
& 2 \\
& 2 \\
& 1 \\
& 2 \\
& 2 \\
& \text { I } \\
& 2 \\
& 2 \\
& \square \\
& \square \\
& \square \\
& \square \\
& \square \\
& \square \\
& \square \\
& 0 \\
& \square \\
& \square \\
& \square \\
& \square \\
& \square \\
& \square \\
& \square \\
& \square \\
& 8.8 \\
& 28 \\
& 2 \\
& 2 \\
& 2 \\
& 2 \\
& 8
\end{aligned}
$$

| $\square$ | $\square$ | $\square$ | $\cdots$ | $\square$ | $\square$ | - | $\square$ | $\square$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | $\square$ | 0 | $\square$ | 0 | 0 | 0 | 0 | 0 |
| 8 | - 2 | 8 | 8 | $\square$ | 2 | 8 | $\square$ | 8 |
| $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | - | $\square$ | $\square$ |
| 0 | 0 | 0 | - 0 | $\square$ | 0 | 0 | 0 | , |
| 2 | - 2 | 2 | 8 | $\square$ | 2 | 2 | $\square$ | 8 |
| $\square$ | $\square$ | - 0 | - | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ |
| 0 | $\square$ | $\square$ | 0 | $\square$ | 0 | 0 | $\square$ | 0 |
| 8 |  |  |  |  |  |  |  |  |


| , | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | - | $\Delta \bullet$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $\square$ | 0 |
| 8 | $\square$ | 2 | 8 | - 2 | 8 |  | 18 | L | 2 |
| - | - | - | - | $\square$ | $\square$ | $\square$ | $\square$ | 0 |  |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| 2 | $\square$ | 2 | 8 | 0 | 2 | 2 | $1 \cdot$ |  | 2 |
| $\square$ | $\square$ | $\square$ | - |  | $\square$ |  | , | , |  |
| 0 | 0 | $\square$ | 0 | $\square$ | $\square$ | 0 | $\square$ | $\square$ | 0 |
|  |  |  |  |  |  |  |  |  |  |

## What are the possible SETs?



## What are the possible SETs?



## What are the possible SETs?



$$
\begin{aligned}
& \square \square \square \square \square \square \square \square \square \square \square \square \square \\
& 0 \square 0 \\
& \text { [8] -8 } \\
& \square \square \\
& \square \\
& \square \\
& \square \\
& \square \\
& \square \\
& \square \square \\
& \square \\
& 2 \square \\
& \text { I } \\
& \text { I } \\
& 8 \\
& \square \\
& \square \\
& \square \\
& \square \\
& \square \\
& \square \\
& 0 \\
& \square \\
& \square \\
& \square \\
& \square \\
& \square \\
& \square \\
& \square \\
& \square \\
& 8.8 \\
& \text { 2 } 8 \\
& 2 \\
& 2 \\
& 2 \\
& 2 \\
& 8
\end{aligned}
$$

$$
\begin{aligned}
& \square \square \square \square \square \square \square \square \square \\
& 0 \square 0 \\
& \square \\
& \square \\
& \cdot \square \\
& \square \\
& \cdot 0 \\
& 2 \\
& -2 \\
& 2 \\
& 2 \\
& \cdot \square \\
& 1 \\
& 2 \\
& \cdot \square \\
& 2 \\
& \square \\
& \square \\
& \square \\
& \square \\
& \square \\
& \square \\
& \square \\
& \square \\
& \square \\
& \square \\
& \cdot 0 \\
& \text { I } \\
& \cdot 0 \\
& \text {. } 8.8 \\
& 2
\end{aligned}
$$

$$
\begin{aligned}
& \square \square \square \square \square \square \square \square \square \\
& 0 \square 0 \\
& \square \\
& \square \\
& 0010 \\
& \square \\
& \cdot \square \\
& 2 \\
& \cdot 8 \\
& 2 \\
& 2 \\
& \text { D } 1 \\
& 2 \\
& \cdot 8 \\
& 2 \\
& \square \\
& \square \\
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& \square \\
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& \square \\
& \square \\
& \cdot 0 \\
& 2 \\
& \text { I } \\
& \cdot 0 \\
& \square \\
& \cdot 0 \\
& \square \\
& \square \\
& \text { (2) } 0 \\
& \cdot 2 \\
& 2 \\
& 2 \\
& \text { - } 1
\end{aligned}
$$

|  | $\square$ | - | - | - | , | 0 |  | - | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\square$ | $\square$ | 0 | 0 | 0 | .00 | 0 |  | 0 | $\square$ |
| 2 | -2 | 2 | 2 | . 2 | 2 | 2 |  | 2 | 2 |
| $\square$ | $\square$ | - | $\cdots$ | - | $\square$ | $\square$ |  | - | , |
| 0 | 0 | 0 | 0 | $\square$ | - 0 | 0 |  | 0 | 0 |
| 8 | $\cdots$ | 2 | 2 | - 2 | 2 | 2 |  | 2 | 2 |
|  | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ | $\square$ |  | $\square$ | 1 |
| 0 | 0 | - | 0 | 0 | - 0 | $\square$ |  | 0 | 0 |
| 2 | [0] | 2 | 2 | $\square$ | 2 | 2 |  | 2 | 2 |

$$
\begin{aligned}
& \square \square \square \square \square \square \square \square \square \\
& 0.00 \\
& 2 \\
& \cdot \square \\
& 2 \\
& 1 \\
& \square \\
& 00 \\
& \square \\
& \square \\
& 2 \\
& \text { I } \\
& -8 \\
& 2 \\
& \square \\
& \text { - } 1 \\
& \text { - } \quad \\
& \square \\
& \square \\
& \square \\
& \square \\
& \square \\
& \square \\
& \cdot 0 \\
& \cdot 0 \\
& 2 \\
& 2 \\
& \cdot \square \\
& \square \\
& \square \\
& \cdot 0 \\
& 1 \\
& \square \\
& \text { 2. } 0 \\
& \text { 2 } 8 \\
& \cdot \square \\
& 2 \\
& 8 \\
& \text {. } 2
\end{aligned}
$$

## What are the possible SETs?



## What are the possible SETs?



## We observed that SETs lie on lines!



## Is this a real thing?!?

## Is this a real thing?!?

Why can we just repeat the grid like that?

## Is this a real thing?!?

Why can we just repeat the grid like that?

Does every SET have to lie on a line? Or is it just the ones that we found?

Why can we just repeat the grid like that?


Why can we just repeat the grid like that?

$$
\begin{aligned}
& \square \square \square \square \square \square \square \square \\
& \square \\
& \square \square
\end{aligned}
$$

Why can we just repeat the grid like that?

$$
\begin{aligned}
& \square \square \square \square \square \square \square \square \square \square \square \\
& 0 \square
\end{aligned}
$$

Why can we just repeat the grid like that?

## 

## 14 is the same as 2

26 is the same as 2


This is called working mod 12. We do usual arithmetic, but treat 12 as 0.

This is called working mod 12. We do usual arithmetic, but treat 12 as 0.


This is called working mod 12.
We do usual arithmetic, but treat 12 as 0.


It makes sense to repeat when we work mod 3!

## Is this a real thing?!?

Why can we just repeat the grid like that?


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## Weird but AMAZING thing about mod 3 arithmetic:

The only possible slopes are $1,0,-1$, or $\infty$

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Weird but AMAZING thing about mod 3 arithmetic:
The only possible slopes are $1,0,-1$, or $\infty$

$$
\begin{aligned}
& \begin{array}{|l|l|l|l|l|l|}
\hline 2 & 2 & 1 & 8 & 8 \\
\hline & & 1 & & 1 & \\
\hline
\end{array} \\
& \square \\
& \square \\
& \square \square
\end{aligned}
$$

Weird but AMAZING thing about mod 3 arithmetic:
The only possible slopes are $1,0,-1$, or $\infty$


## Does every SET have to lie on a line? Or is it just the ones that we found?

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## Does every SET have to lie on a line? Or is it just the ones that we found?



SET

## Does every SET have to lie on a line? Or is it just the ones that we found?



## Does every SET have to lie on a line? Or is it just the ones that we found?


slopes are 1, 1, 1

## Does every SET have to lie on a line? Or is it just the ones that we found?


slopes are 1, 1, 1


## Does every SET have to lie on a line? Or is it just the ones that we found?



SET
slopes are 1, 1, 1


## NOT A SET

slopes are 0, 1, -1
If 3 points lie on a line, then all pairs have the same slope If 3 points do not lie on a line, then all pairs have different slopes

## Does every SET have to lie on a line? Or is it just the ones that we found?

## Does every SET have to lie on a line? Or is it just the ones that we found?

If 3 points lie on a line, then all pairs have the same slope If 3 points do not lie on a line, then all pairs have different slopes

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If 3 points lie on a line, then all pairs have the same slope If 3 points do not lie on a line, then all pairs have different slopes

Weird but AMAZING thing about mod 3 arithmetic:
The only possible slopes are $1,0,-1$, or $\infty$

## Does every SET have to lie on a line? Or is it just the ones that we found?

If 3 points lie on a line, then all pairs have the same slope If 3 points do not lie on a line, then all pairs have different slopes

Weird but AMAZING thing about mod 3 arithmetic:
The only possible slopes are $1,0,-1$, or $\infty$
If you pick 3 different numbers from 1,0,-1, or $\infty$, you must end up with a 0 or $\infty$ !

## Does every SET have to lie on a line? Or is it just the ones that we found?

If 3 points lie on a line, then all pairs have the same slope If 3 points do not lie on a line, then all pairs have different slopes

Weird but AMAZING thing about mod 3 arithmetic:
The only possible slopes are $1,0,-1$, or $\infty$
If you pick 3 different numbers from 1,0,-1, or $\infty$, you must end up with a 0 or $\infty$ !


## Is this a real thing?!?

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Does every SET have to lie on a line? Or is it just the ones that we found?

This is a real thing!?!

