Strategic Game Theory

Games!

Board games!

Sport games!

Video games!

More video games!

Politics and Finance

Today: play a series of games, see if we can figure out how to win!

Game 1: 21

- Two players take turns counting
- Start from "1"
- Each player can count at most three numbers
- ▶ First player that says "21" wins

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- Two players take turns counting
- Start from "1"
- Each player can count at most three numbers
- ▶ First player that says "21" wins
 - Alice: 1, 2, 3
 - Bob: 4, 5
 - Alice: 6, 7
 - Bob: 8, 9, 10
 - Alice: 11
 - Bob: 12, 13, 14
 - Alice: 15, 16
 - Bob: 17, 18, 19
 - Alice: 20, 21
 - Alice wins

Let's try it!

What's the trick here?



▶ Same rules, but whoever first counts to "22" wins

Same rules, but whoever first counts to "22" wins
What about "23"?

- Same rules, but whoever first counts to "22" wins
- What about "23"?
- What about "24"? Who should go first?

- Same rules, but whoever first counts to "22" wins
- What about "23"?
- What about "24"? Who should go first?
- ▶ What about "2023"?

Same rules, but now each player can count up to four numbers each time

- Same rules, but now each player can count up to four numbers each time
- What about five numbers each time?

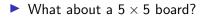
Final test for Game 1

Each player can count up to nine numbers, first one to "2023" wins

Game 2: Grid game

- ▶ 4 × 5 board
- Start by placing a rock at the bottom left
- Each turn, a player can move the rock directly above, directly to the right, or diagonally up and above
- Whoever moves the rock at the top right corner wins

Game 2 modification



Game 2 modification

- What about a 5×5 board?
- ▶ What about a 8 × 8 board?

Game 2 modification

- ▶ What about a 5 × 5 board?
- What about a 8 × 8 board?
- ▶ What about a 100 × 100 board?

Game 3: Piles of stones

- Two piles of stones: 5, 7
- Each turn, a player can pick a pile, and take any number of stones from it
- Player who takes the last stone wins

Game 3 modification

- ► Three piles of stones: 3, 5, 7
- Player who takes the last stone wins