Induction

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Here is an informal statement of the principle of induction.

Theorem 1. *Let* X *be a subset of the natural numbers,* $X \subset \mathbb{N}$*. Suppose* $1 \in X$ *and if* $n \in X$ *then* $n+1 \in X$*. Then* $X = \mathbb{N}$*.*

Proof. Every non-empty subset of $\mathbb{N}$ has a least element. Let $Y = \mathbb{N} - X$. If $X \neq \mathbb{N}$ then $Y \neq \emptyset$ and hence has a least element m. We know $1 \in X$. So $m > 1$ and $m - 1 \notin Y$, since m is the least element in Y. Hence $m - 1 \in X$. But then $m \in X$, by our assumption. This can't be since $m \in Y$, so $m \notin X$. Conclusion: $Y = \emptyset$ and $X = \mathbb{N}$. □